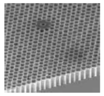
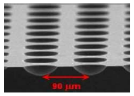
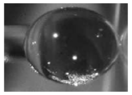
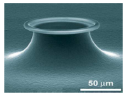
    

Strongly correlated phenomena in cavity QED

Fernando G.S.L. Brandão[1,2]

Michael J. Hartmann[1,2]   Martin B. Plenio[1,2]

[1]Institute for Mathematical Sciences, Imperial College London
[2]QOLS, Blackett Laboratory, Imperial College London

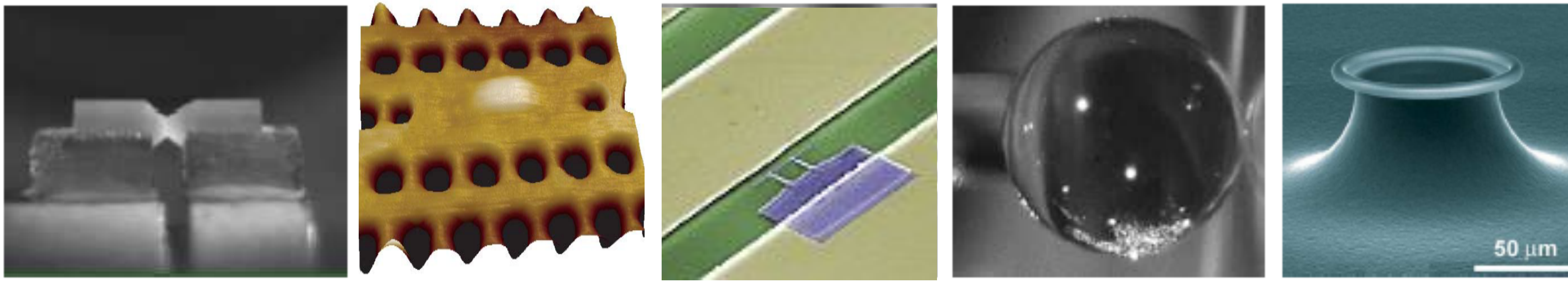
London, 04/05/2007

Imperial College London

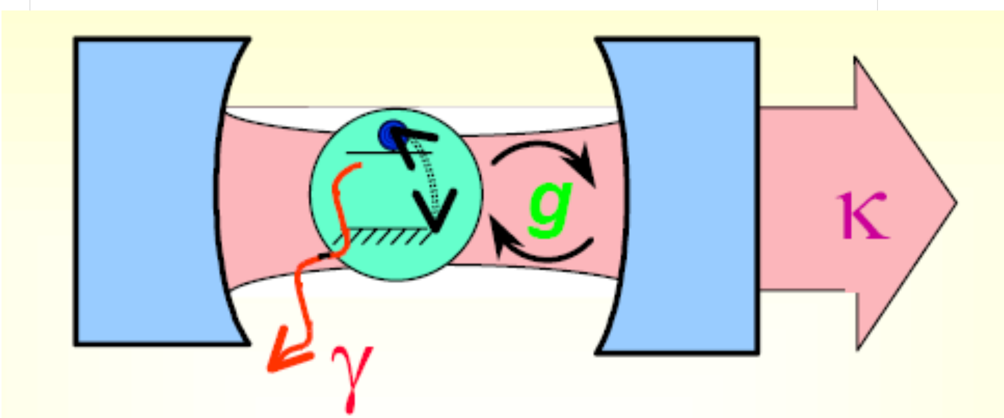
Institute for Mathematical Sciences

QI IC

Cavity QED systems

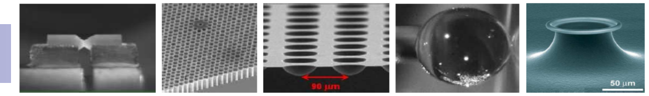


Non-trivial joint dynamics for atoms and photons

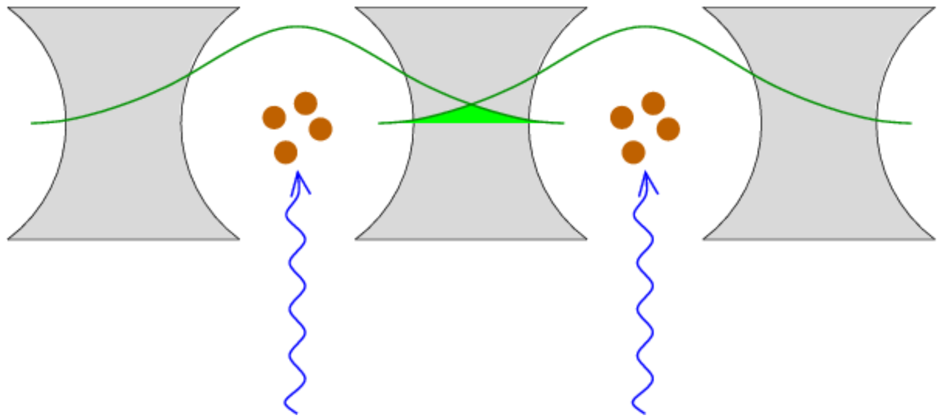


Strong Coupling:

$$g >> \gamma, \kappa$$

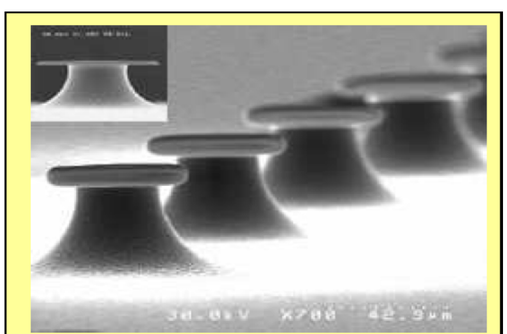


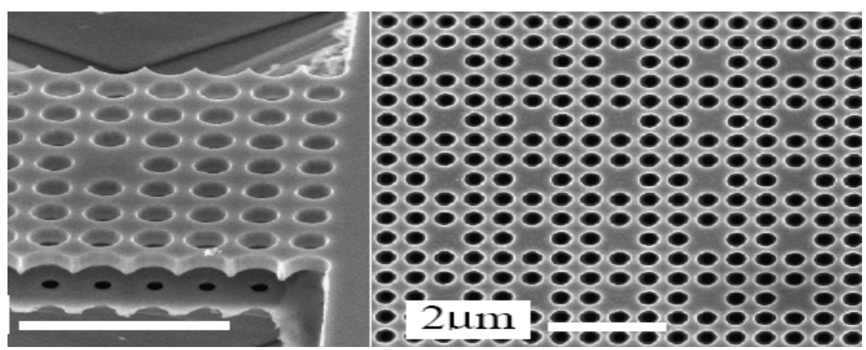
Array of coupled cavities

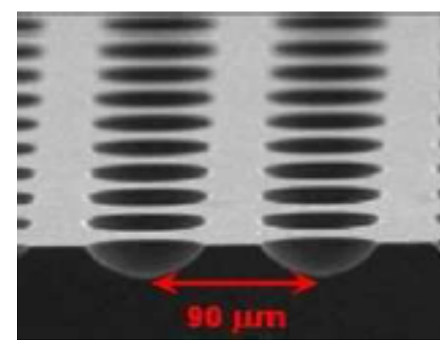


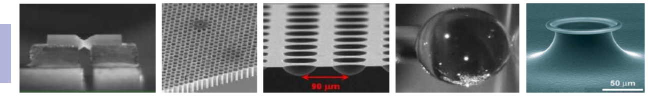
Atoms in different cavities can “talk” to each other mediated by the photons

Photons in the same cavity can “talk” to each other mediated by the atoms









Summary

- Photon nonlinearities

  EIT-based schemes

  Stark-shift based scheme

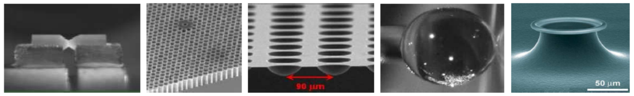
- Bose-Hubbard models

  Polaritons in coupled array of cavities

  The photonic limit

- Spin Chains

  Heisenberg model (XYZ)



Summary

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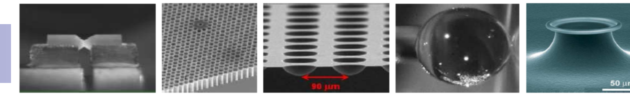


Photon-Photon interactions

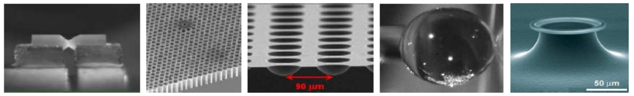
- Kerr-type nonlinear interaction:

$$H_{kerr} = \eta a^{\dagger} a a^{\dagger} a$$

- Several applications

Photon blockade	Imamoğlu *et at*, PRL **79**, 1467 (1997)
nonlinear optics	Boyd, *Nonlinear Optics,* (1992)
Quantum nondemolition measurents	Imoto *et al ,* PRA **32,** 2287 (1985)
Optical quantum computing	Turchette *et al*, PRL **75**, 4710 (1995)

etc…



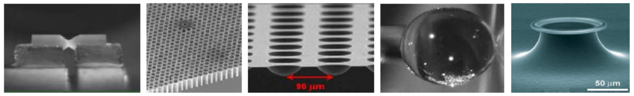
Photon-Photon interactions

- Kerr-type nonlinear interaction:

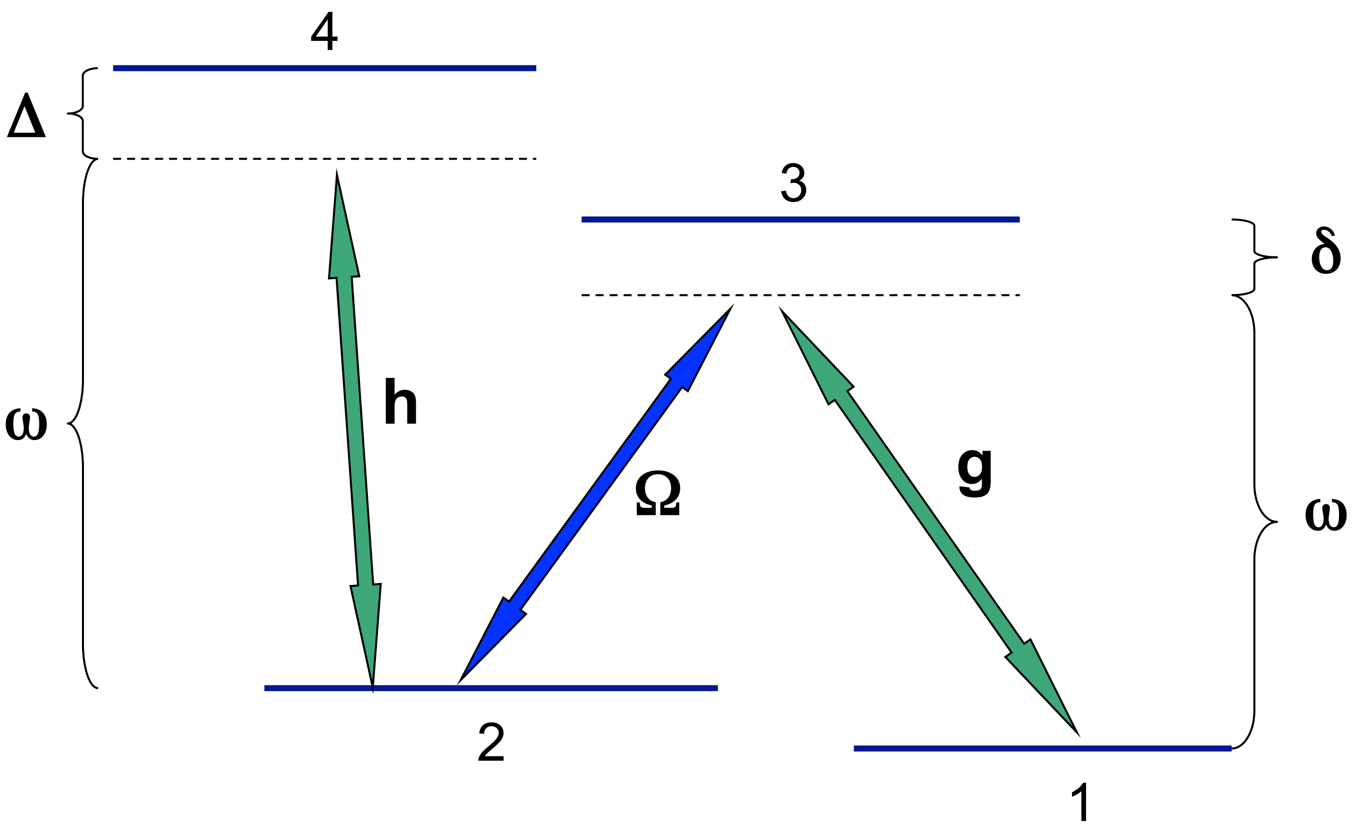
$$H_{kerr} = \eta a^{\dagger} a a^{\dagger} a$$

Natural Kerr interactions are far too small…

$$\eta <<<< \kappa, \gamma$$



Electromagnetically Induced Transparency nonlinearities



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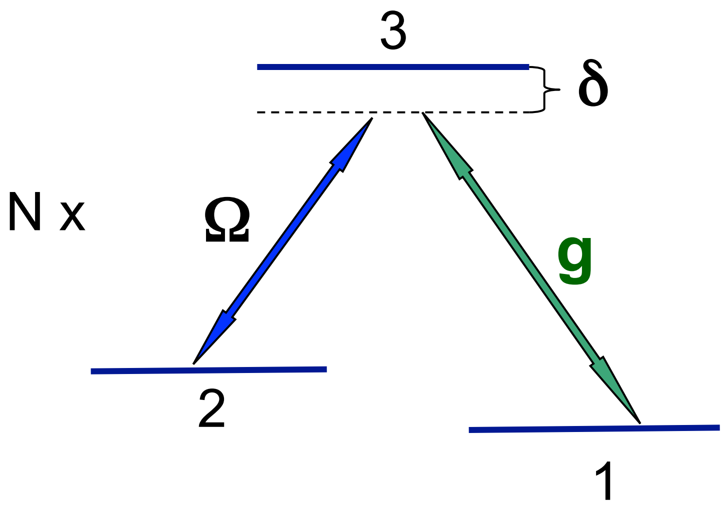
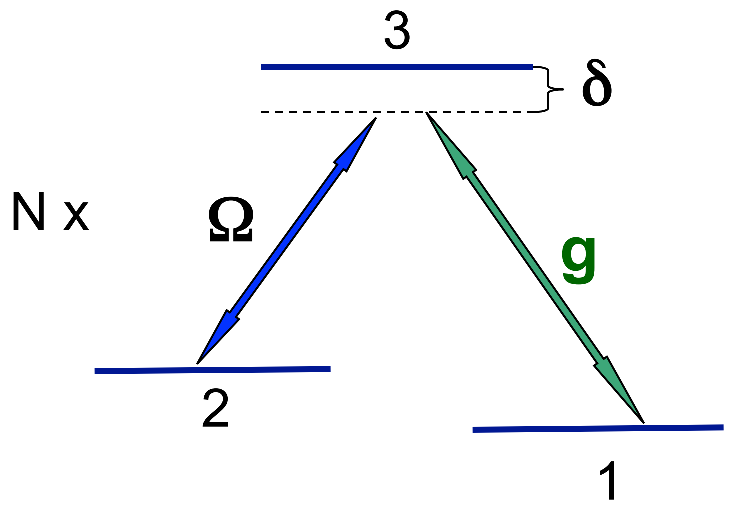
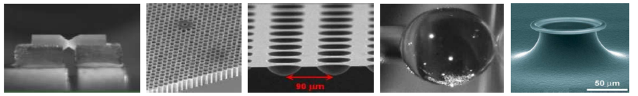
Electromagnetically Induced Transparency nonlinearities



Electromagnetically Induced Transparency nonlinearities

$$H = \mu_0 p_0^\dagger p_0 + \mu_+ p_+^\dagger p_+ + \mu_- p_-^\dagger p_-$$



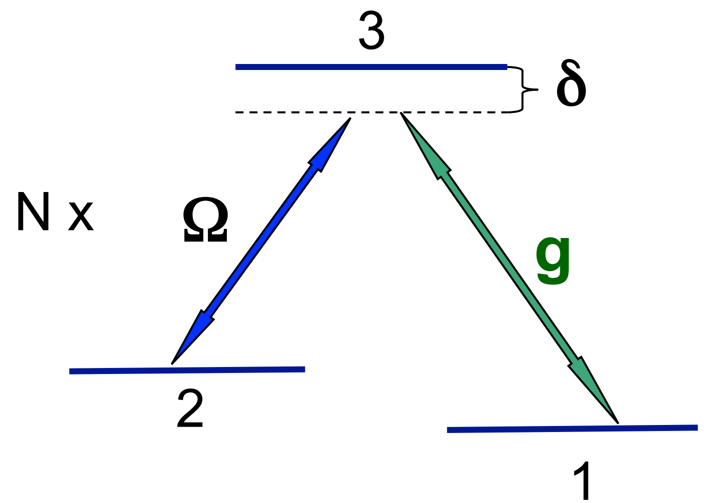


Electromagnetically Induced Transparency nonlinearities

$$H = \mu_0 \, p_0^\dagger p_0 + \mu_+ \, p_+^\dagger p_+ + \mu_- \, p_-^\dagger p_-$$

$$p_0^\dagger = \frac{1}{\sqrt{N g^2 + \Omega^2}} \left(\sqrt{N} g \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |2_j\rangle\langle 1_j| - \Omega \, a^\dagger \right)$$

$\Omega \gg \sqrt{N} g$: photonic excitation



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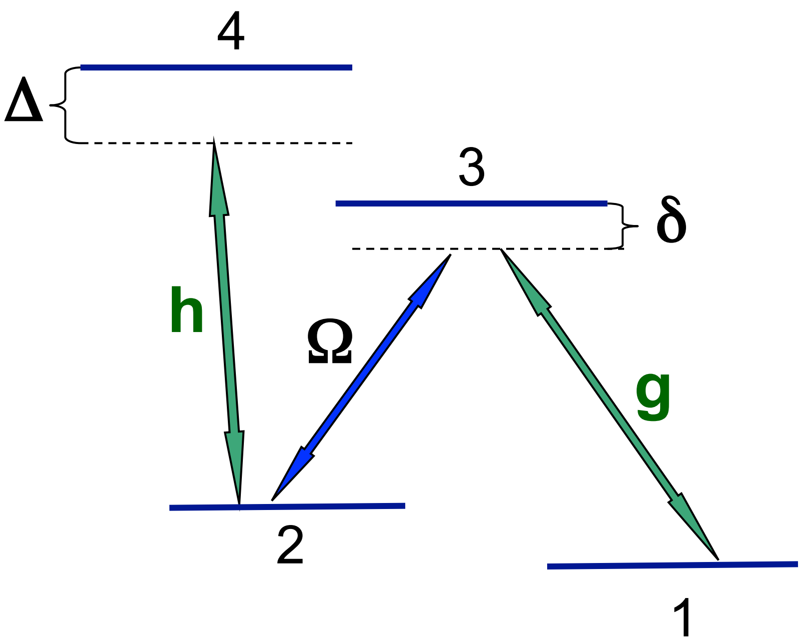
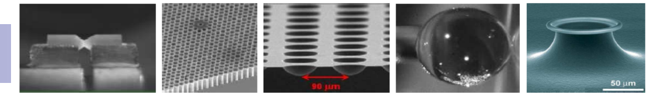
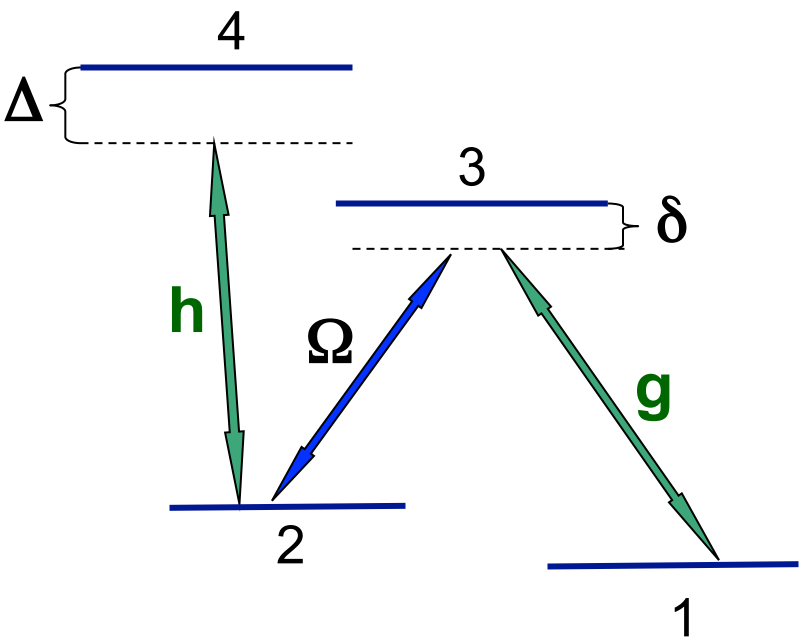


$|h|, |\Delta| \ll$ "energy differences between different polariton species"



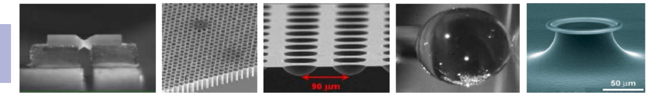
Electromagnetically Induced Transparency nonlinearities



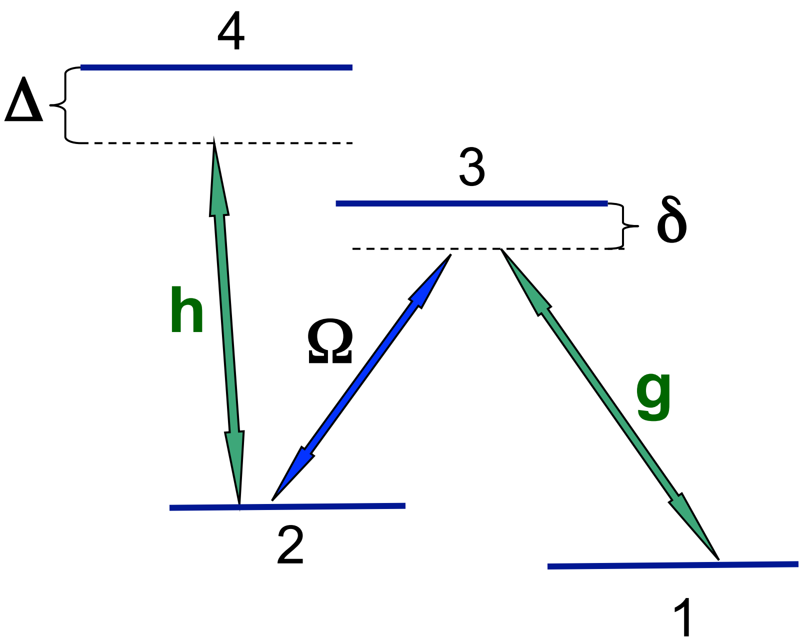
$|h|\,,\ |\Delta| \ll$ "energy differences between different polariton species"

$$H_4 = -\frac{hg}{\Omega}\left(\sum_{k=1}^{N} |4_k\rangle\langle 1_k| p_0^2 + h.c.\right)$$

Only dark state polaritons p_0 couple to level 4!



Electromagnetically Induced Transparency nonlinearities



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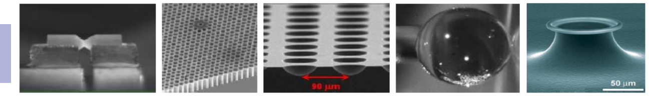
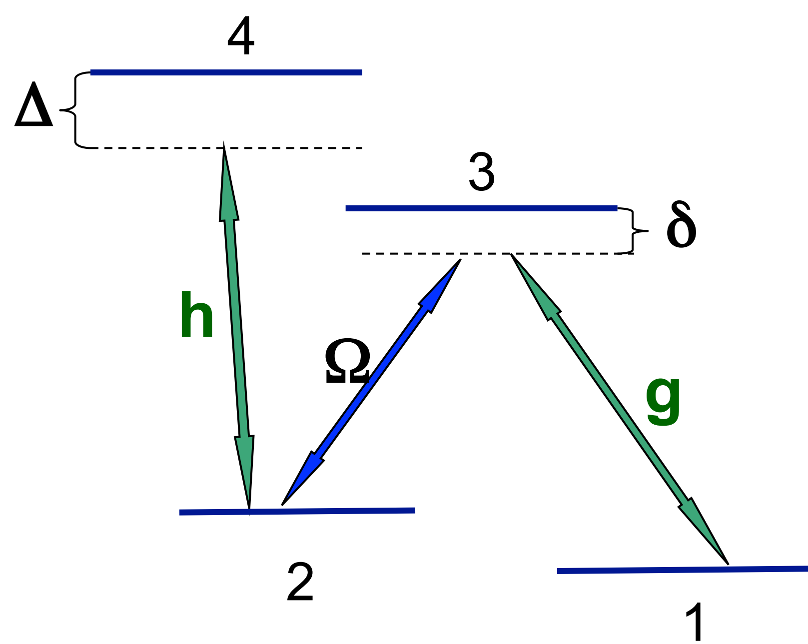
$$H_4 = -\frac{hg}{\Omega}\left(\sum_{k=1}^{N} |4_k\rangle\langle 1_k| p_0^2 + h.c.\right)$$

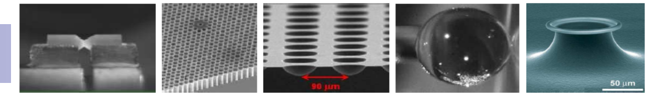
$$\frac{\sqrt{N}gh}{\Omega} \ll \Delta$$



Electromagnetically Induced Transparency nonlinearities

$$H_{eff} = -\underbrace{\frac{\sqrt{N}g}{\Omega}}_{\ll 1}\underbrace{\frac{\sqrt{N}gh}{\Omega\Delta}}_{\ll 1} h a^\dagger a^\dagger a a$$



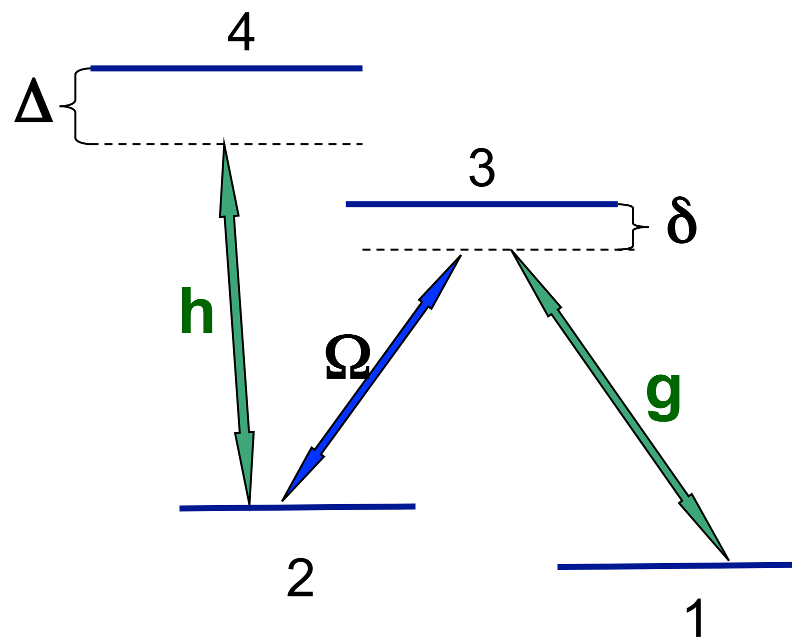


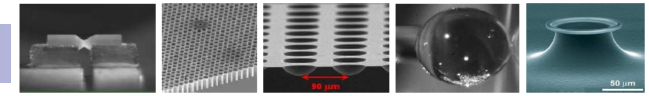
Electromagnetically Induced Transparency nonlinearities

$$H_{eff} = -\underbrace{\frac{\sqrt{N}g}{\Omega}}_{\ll 1}\underbrace{\frac{\sqrt{N}gh}{\Omega\Delta}}_{\ll 1} h a^{\dagger}a^{\dagger}aa$$

We didn't assume:

$\Delta >> h$





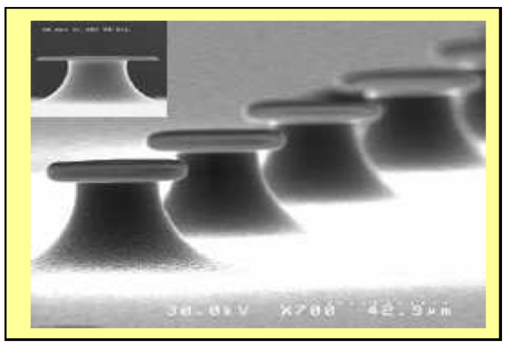
Electromagnetically Induced Transparency nonlinearities

$$H_{eff} = -\underbrace{\frac{\sqrt{N}g}{\Omega}}_{\ll 1}\underbrace{\frac{\sqrt{N}gh}{\Omega\Delta}}_{\ll 1} h a^\dagger a^\dagger a a$$

Example: **Toroidal Microcavities**

Spillane et al, PRA **71**, 013817 (2005)

Aoki et al, Nature **443** 671 (2006)



$$\frac{gh}{\Delta\Omega} = 0.1, \quad \frac{Ng}{\Omega} = 0.1, \quad h = 2.5 \times 10^9 s^{-1}, \quad \kappa = 4 \times 10^4 s^{-1}$$

Electromagnetically Induced Transparency nonlinearities

$$H_{eff} = -\underbrace{\frac{\sqrt{N}g}{\Omega}}_{\ll 1}\underbrace{\frac{\sqrt{N}gh}{\Omega\Delta}}_{\ll 1} h a^{\dagger}a^{\dagger}aa$$

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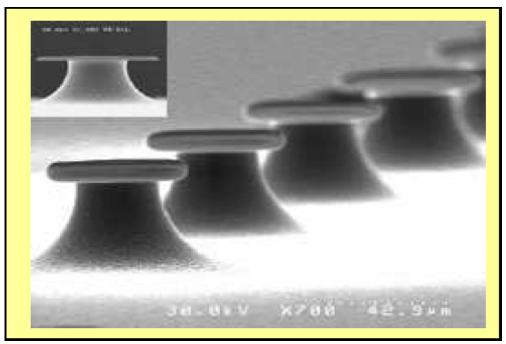
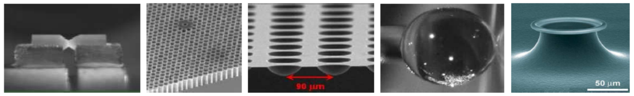


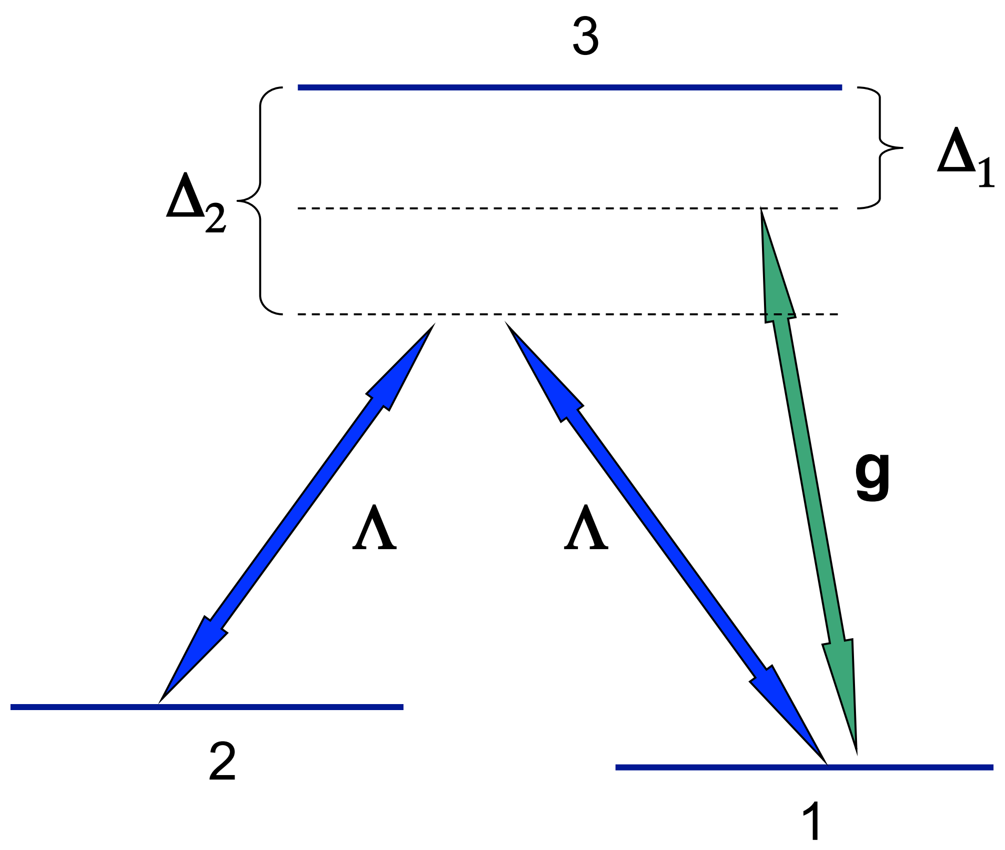
$$\frac{Ng}{\Omega}\frac{gh}{\Omega\Delta}h \approx 625\kappa$$

$$\frac{gh}{\Delta\Omega} = 0.1, \quad \frac{Ng}{\Omega} = 0.1, \quad h = 2.5\times 10^{9}s^{-1}, \quad \kappa = 4\times 10^{4}s^{-1}$$



Could we find a simpler set-up producing a nonlinearity comparable with the EIT one?

A.C. Stark shift nonlinearity



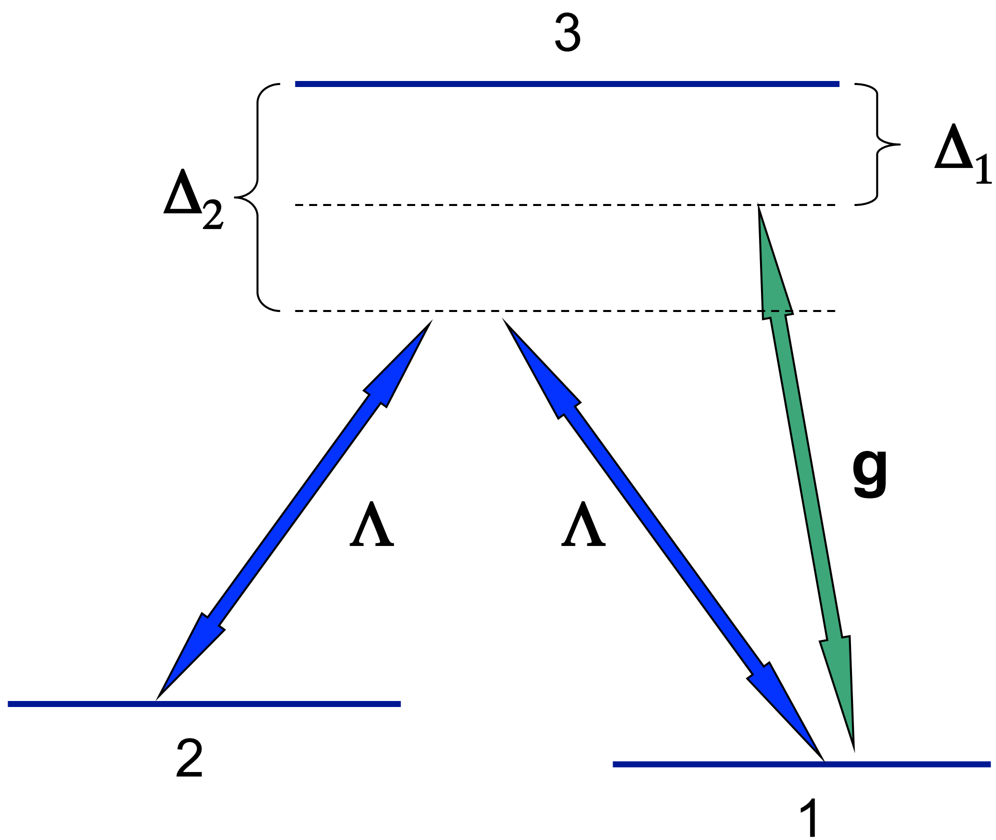
A.C. Stark shift nonlinearity

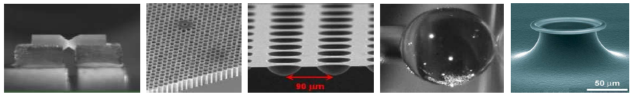
Dispersive regime:

$$\Delta_1 >> g$$

$$\Delta_2 >> \Lambda$$

$$|\Delta_1 - \Delta_2| >> g, \Lambda$$





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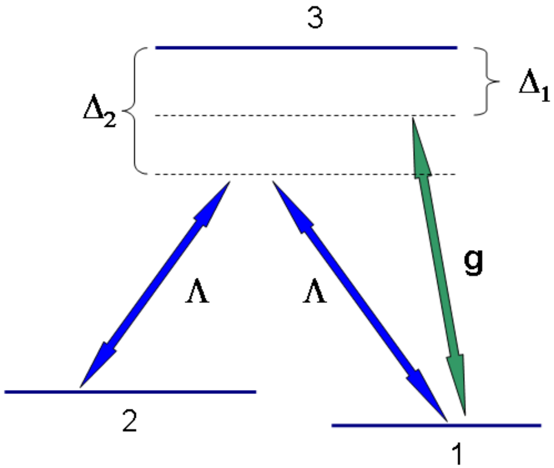
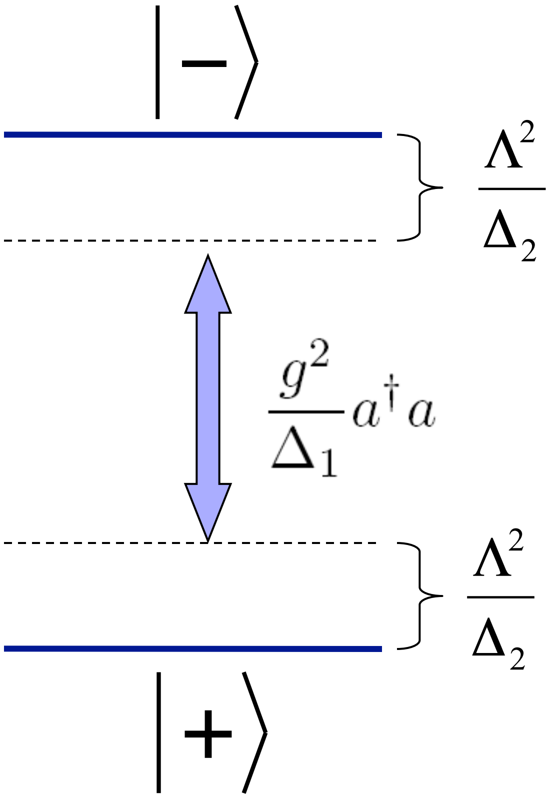
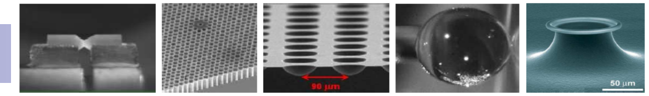


$$|+\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}, |-\rangle = \frac{|1\rangle - |2\rangle}{\sqrt{2}}$$



$$H = \frac{\Lambda^2}{\Delta_2}(|+\rangle\langle+| - |-\rangle\langle-|) + \frac{g^2}{\Delta_1} a^\dagger a(|+\rangle\langle-| + |-\rangle\langle+|)$$



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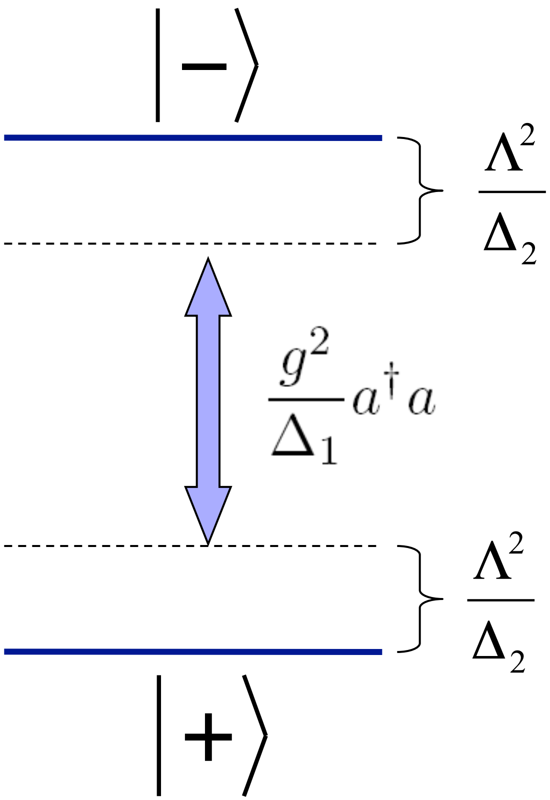
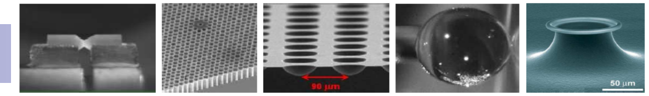


Dispersive regime:

$$\Theta := \frac{2\Lambda^2}{\Delta_2} >> \frac{g^2}{\Delta_1}$$



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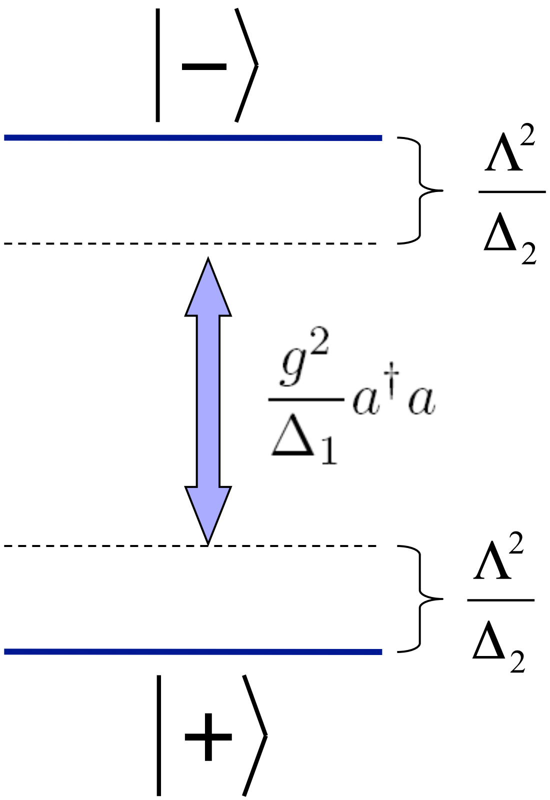


Dispersive regime:

$$\Theta := \frac{2\Lambda^2}{\Delta_2} >> \frac{g^2}{\Delta_1}$$

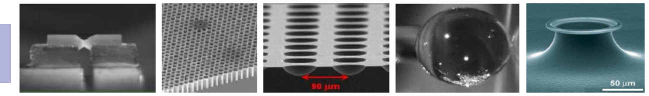
$$H_{eff} = \frac{\left(\frac{g^2}{\Delta_1}a^\dagger a\right)^2}{\Theta}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$$

A.C. Stark shift nonlinearity

$$H_{kerr} = -\underbrace{\frac{g}{\Delta_1}}_{\ll 1}\underbrace{\frac{g^2}{\Theta\Delta_1}}_{\ll 1} g a^\dagger a a^\dagger a$$

-Same strength as EIT scheme

- One level less



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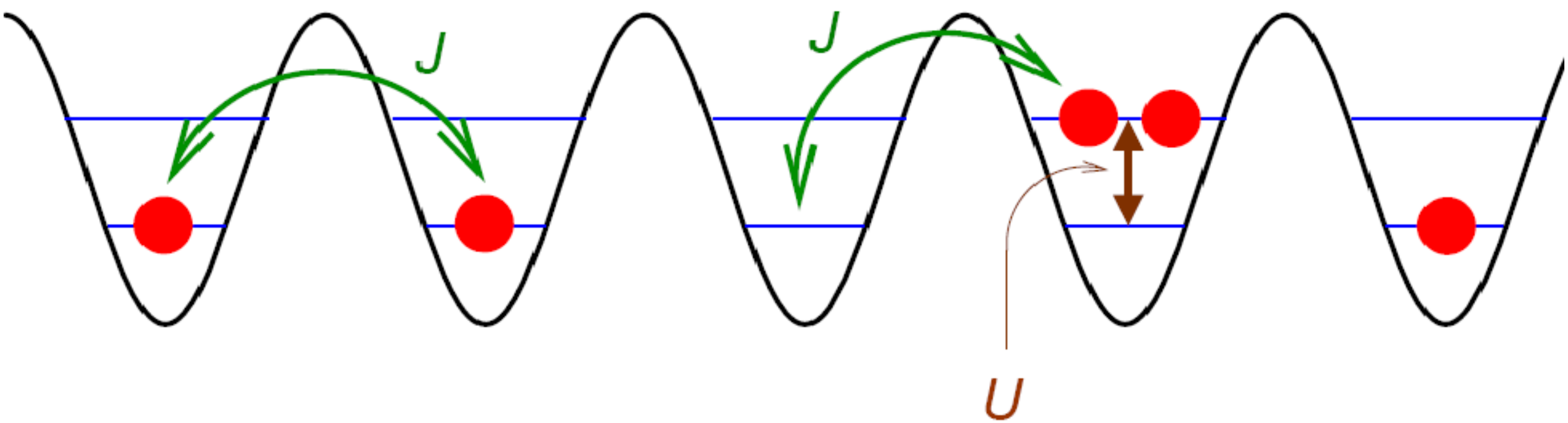
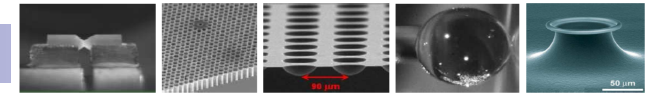
Bose Hubbard Model

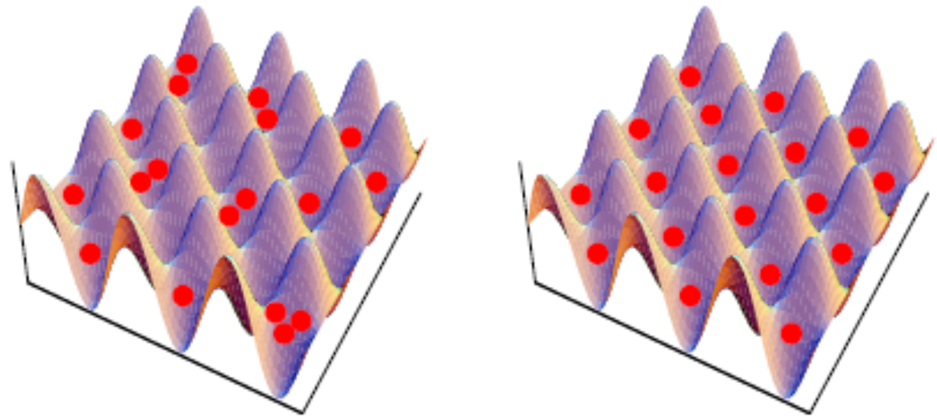


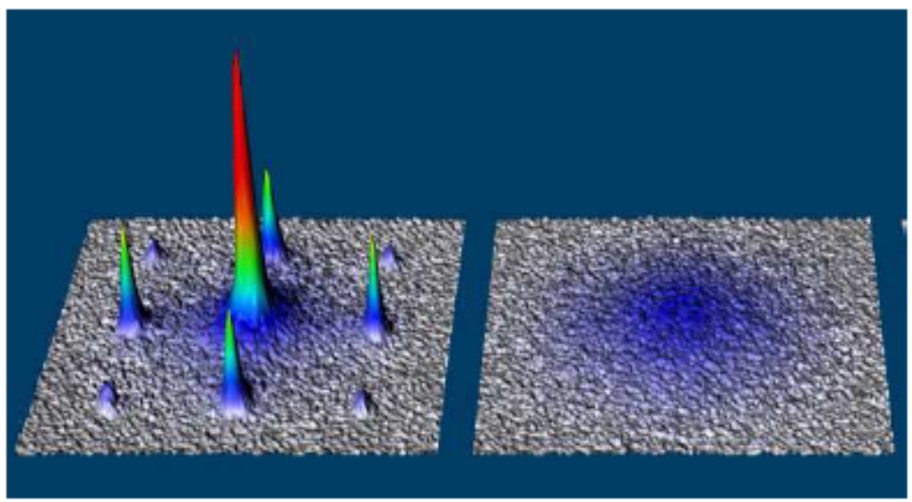
$$H = J\sum_{i=1}^{N}\left(b_i^\dagger b_{i+1} + b_i b_{i+1}^\dagger\right) + U\sum_{i=1}^{N} b_i^\dagger b_i \left(b_i^\dagger b_i - 1\right)$$

Fisher et al, PRB **40**, 546 (1989)



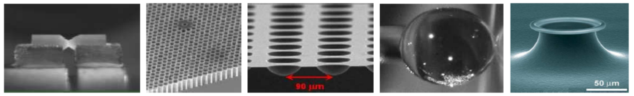
Cold atoms in Optical Lattices





Jaksch et al, PRL **81**, 3108 (1998)

Greiner et al, Nature 415, **39** (2002)



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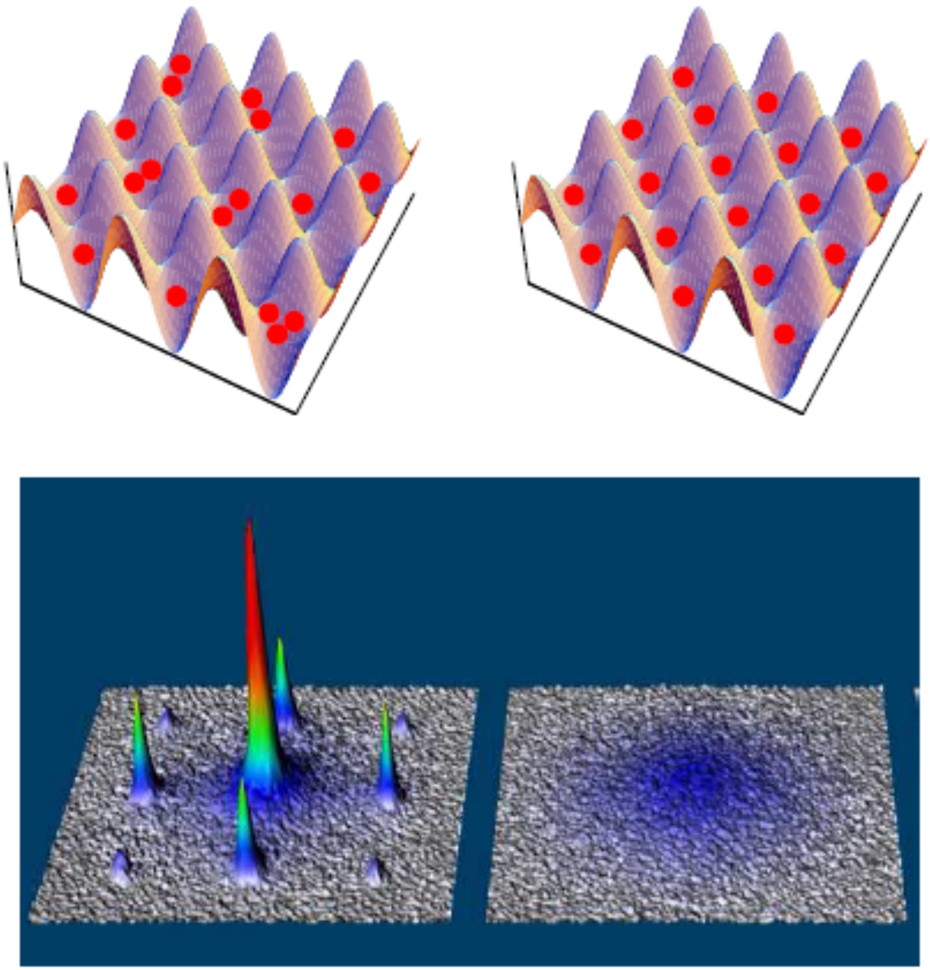

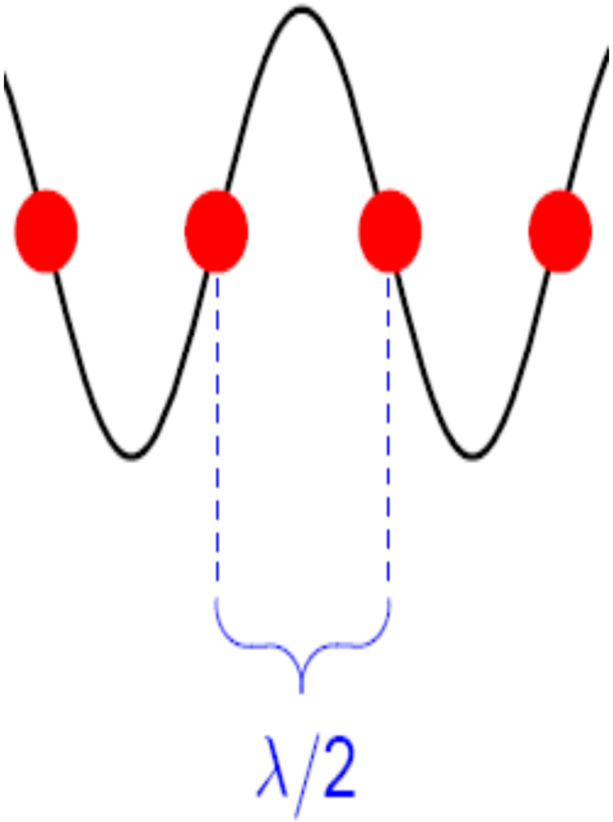

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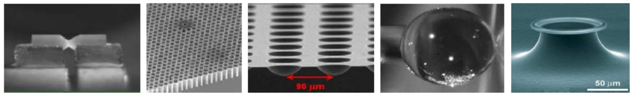
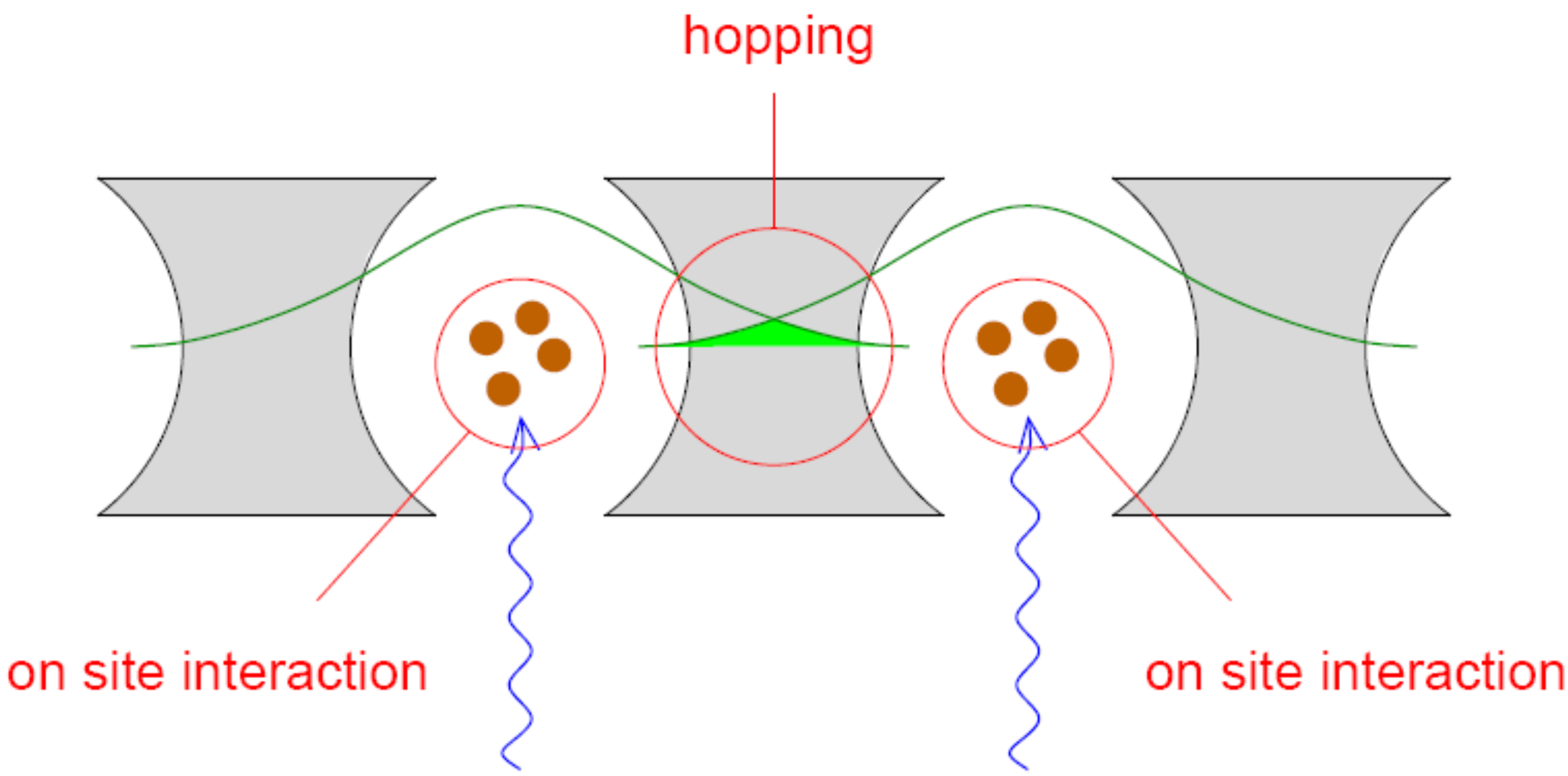
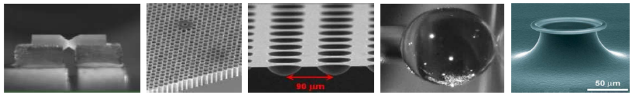


The set-up





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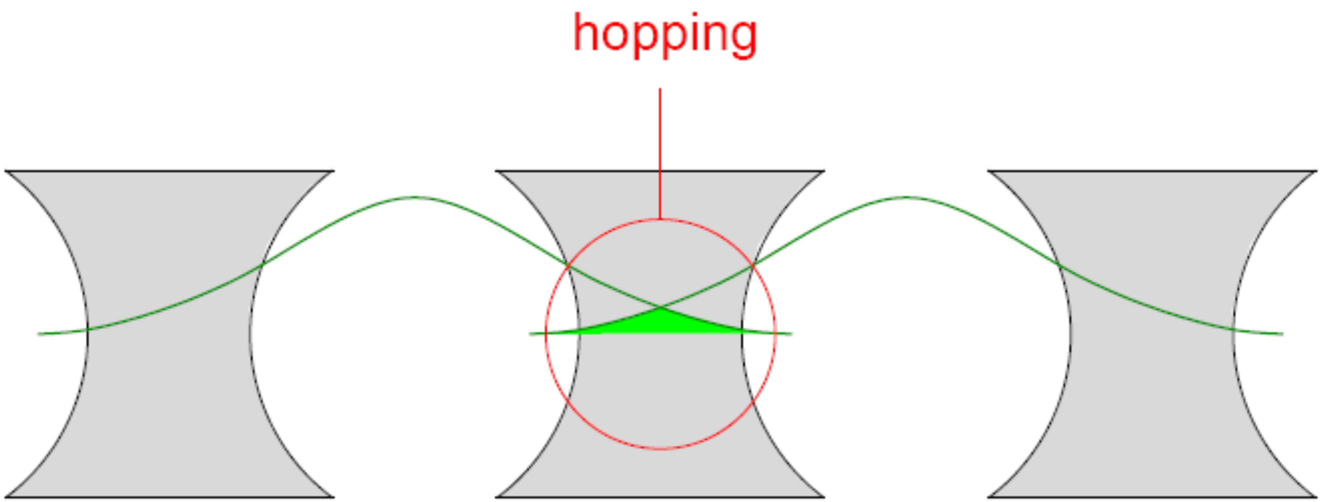
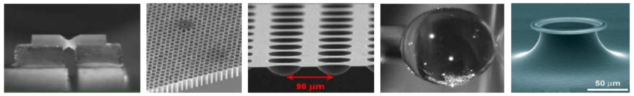


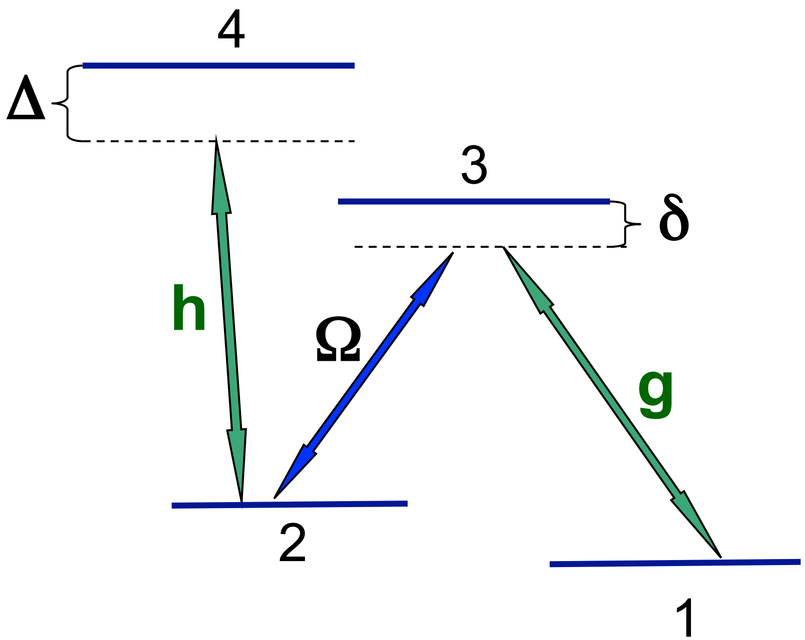
Photons can hope from one cavity to a neighbouring one

$$\mathcal{H} = \omega \sum_j \left(a_j^\dagger a_j + \frac{1}{2} \right) + 2\omega\alpha \sum_j \left(a_j^\dagger a_{j+1} + \text{h.c.} \right)$$

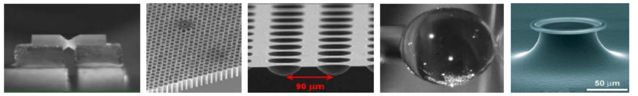
Yariv et al, Optics Lett. **24**, 711 (1999)



The polaritonic case



$$p_0^\dagger = \frac{1}{\sqrt{N g^2 + \Omega^2}} \left(\sqrt{N} g \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |2_j\rangle\langle 1_j| - \Omega a^\dagger \right)$$

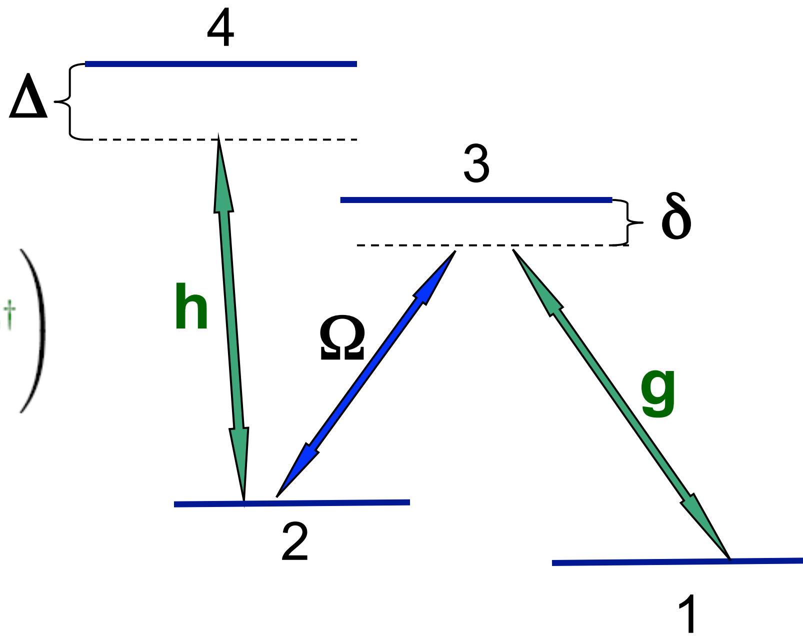


The polaritonic case

$$H_{\text{eff}} = U \sum_j p_j^\dagger p_j \left(p_j^\dagger p_j - 1\right) + J \sum_j \left(p_j^\dagger p_{j+1} + p_j p_{j+1}^\dagger\right)$$

$$U = -\frac{h^2}{\Delta} \frac{N g^2 \Omega^2}{\left(N g^2 + \Omega^2\right)^2} \qquad J = \frac{\Omega^2}{N g^2 + \Omega^2} 2 \omega \alpha$$

$$p_0^\dagger = \frac{1}{\sqrt{N g^2 + \Omega^2}} \left(\sqrt{N} g \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |2_j\rangle\langle 1_j| - \Omega a^\dagger \right)$$



The polaritonic case

spontaneous emission from levels 3 and 4 and cavity decay via photon part lead to effective decay rate Γ for the polaritons

assume that hopping J can be stronger than deacy Γ

$\Rightarrow$ maximize U/Γ

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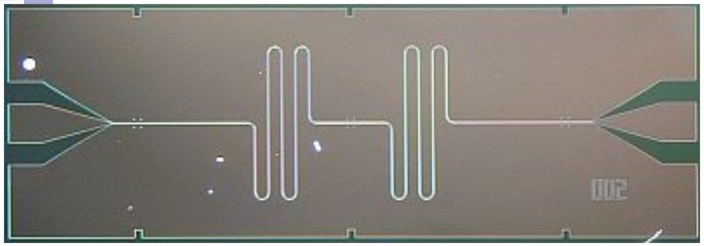
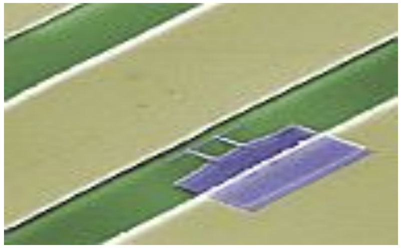
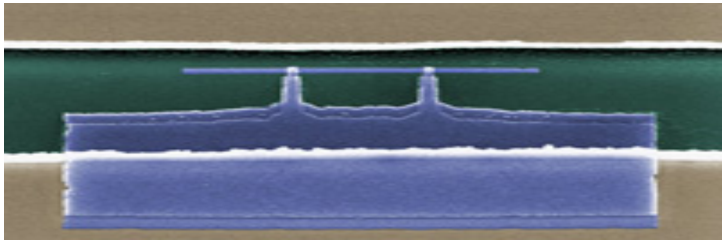
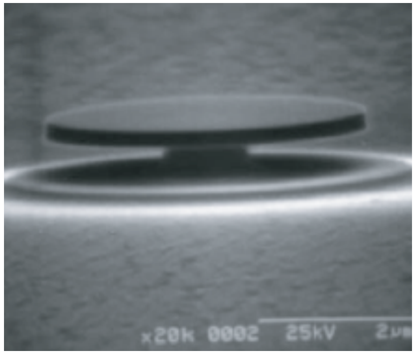
operating point, i.e. value of Ω, can be optimized for each cavity:

- $\Gamma_{SE} \gg \Gamma_C \rightarrow$ make polariton photonic $\rightarrow \sqrt{N}g \ll \Omega$
- $\Gamma_{SE} \ll \Gamma_C \rightarrow$ make polariton atomic $\rightarrow \sqrt{N}g \gg \Omega$









	$g^2/\gamma\kappa$ real	$g^2/\gamma\kappa$ pred.
Fabry-Perot:	160	5×10^3
Photonic bgc:	10	5.5×10^5
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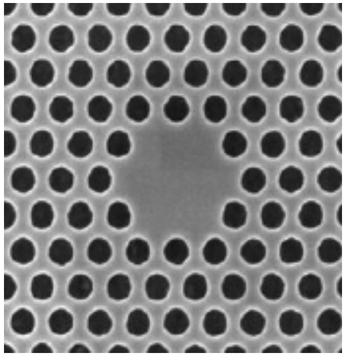
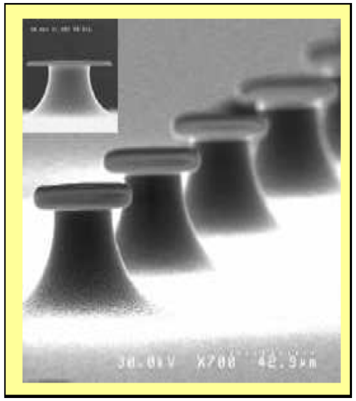
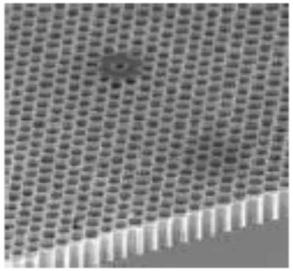
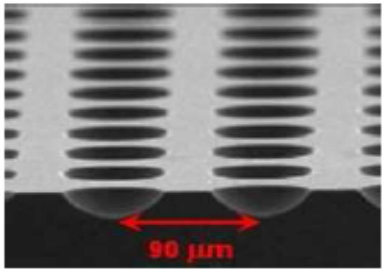


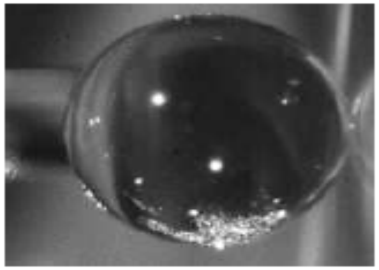


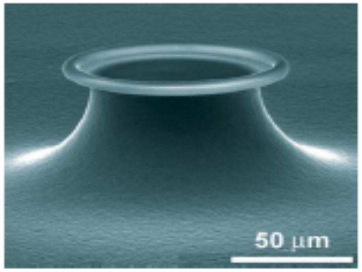
Spillane et al, PRA 2005

Soda et al, Nature Materials 2005

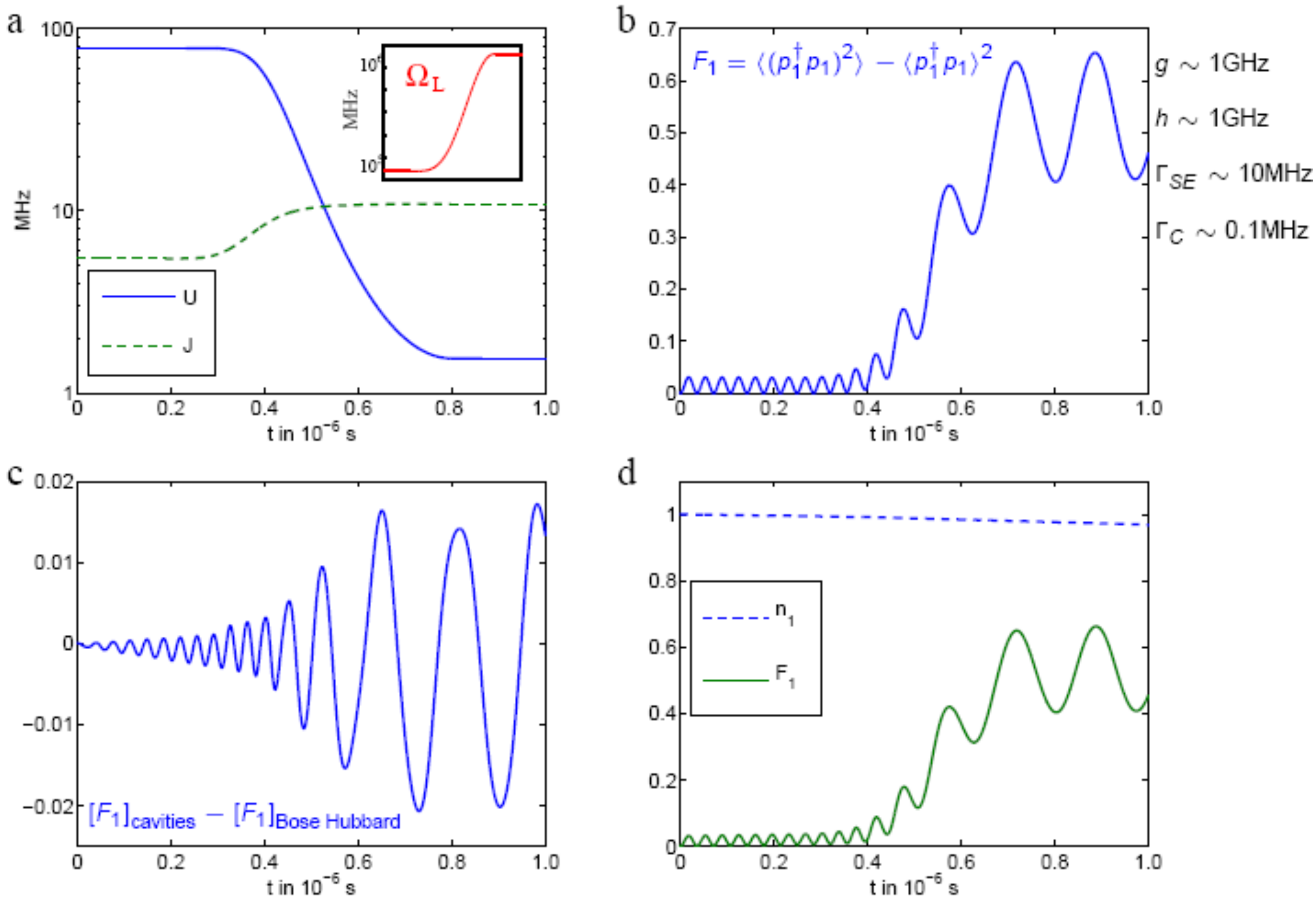


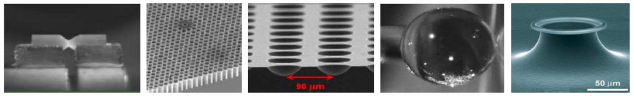






The polaritonic case





The photonic case

$$\mathcal{H}_{eff} = U \sum_j a_j^\dagger a_j (a_j^\dagger a_j - 1) + J \sum_j (a_j^\dagger a_{j+1} + a_j a_j^\dagger)$$

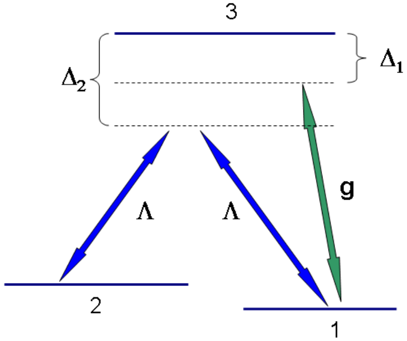
$$J = 2\omega\alpha$$

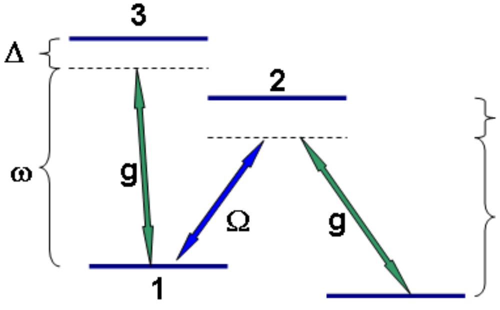
$$U = -\underbrace{\frac{\sqrt{N}g}{\Delta_1}}_{\ll 1}\underbrace{\frac{\sqrt{N}g^2}{\Theta\Delta_1}}_{\ll 1} g$$

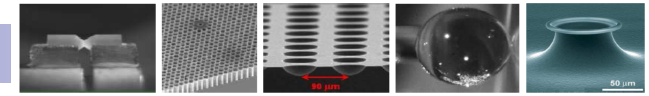
$$U = -\underbrace{\frac{\sqrt{N}g}{\Omega}}_{\ll 1}\underbrace{\frac{\sqrt{N}gh}{\Omega\Delta}}_{\ll 1} h$$

a.c. Stark shift nonlinearity

EIT nonlinearity







The photonic case

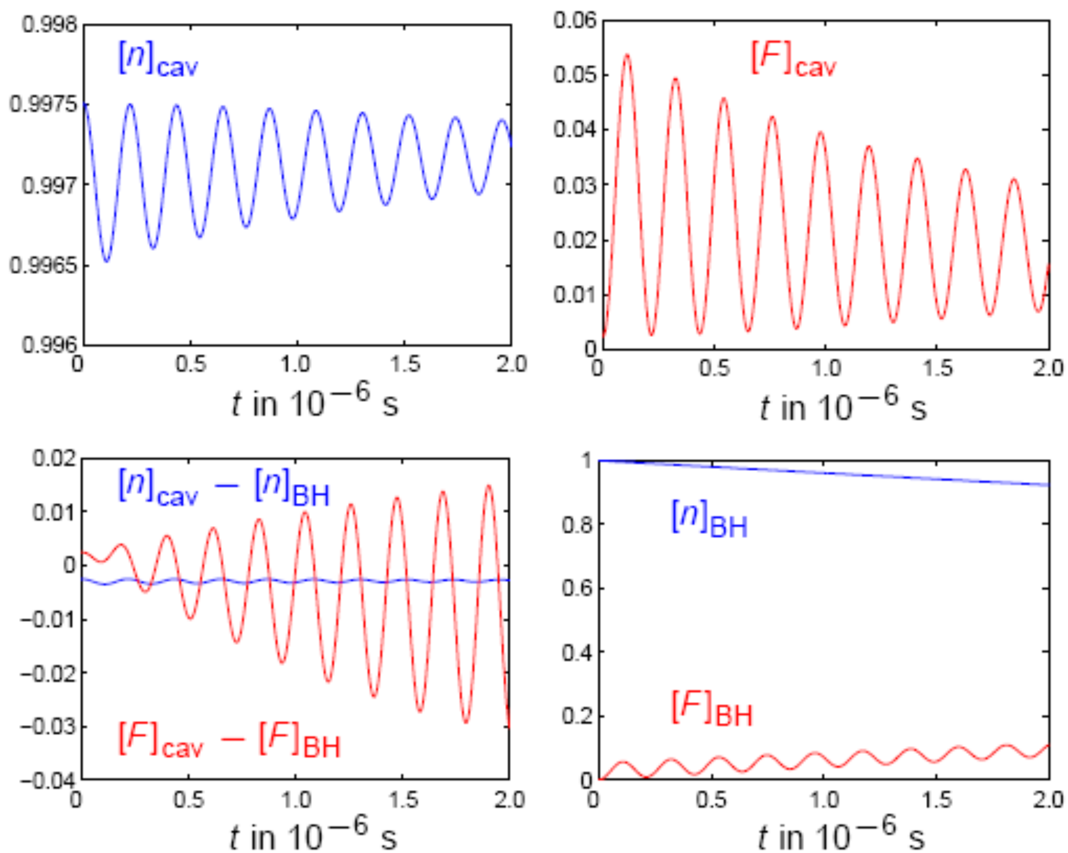
$$\Omega = 20\sqrt{N}\,g$$

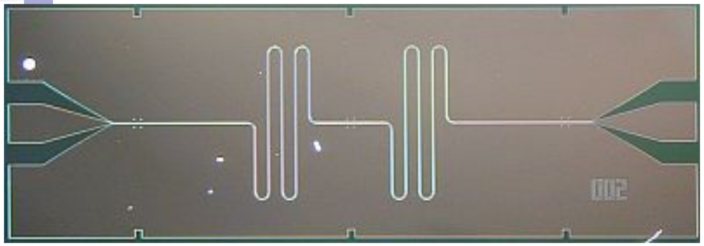
$$n = \langle a^\dagger a \rangle$$

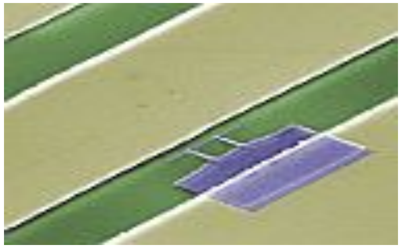
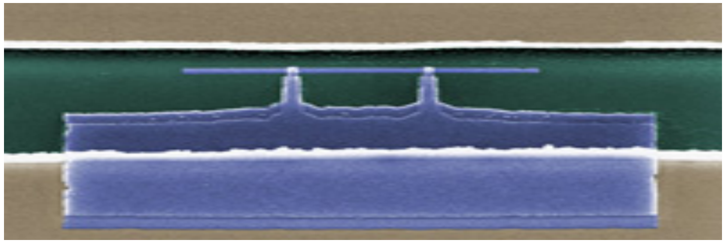
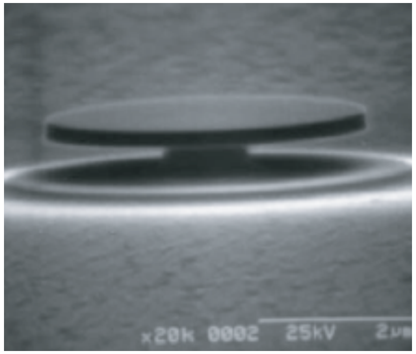
$$F =$$
$$= \langle (a^\dagger a)^2 \rangle - \langle a^\dagger a \rangle^2$$

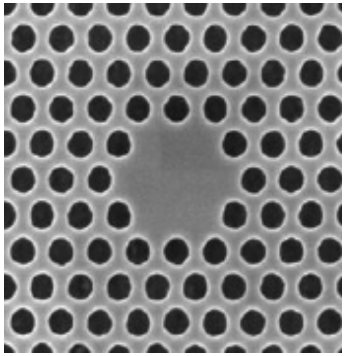
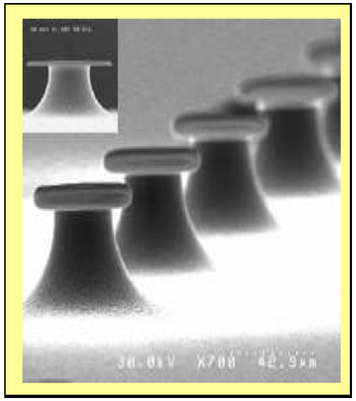
$J \sim 10^6 \mathrm{s}^{-1}$

$U \sim 10^7 \mathrm{s}^{-1}$





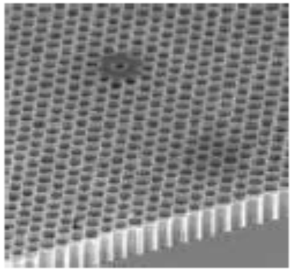
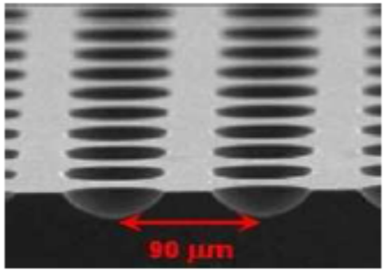




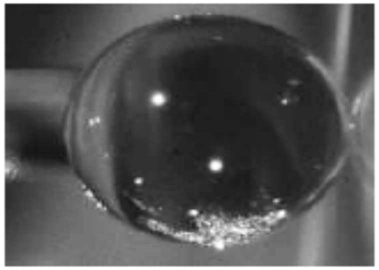
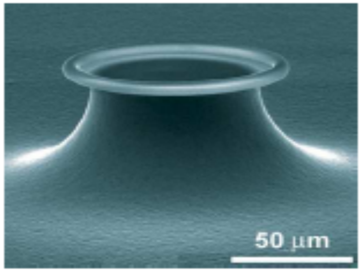




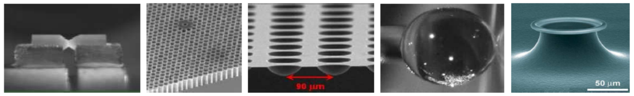
	g/κ real	g/κ pred.
Fabry-Perot:	2.6	10
Photonic bgc:	0.1	4×10^3
MCs @ Imperial:	0.8	?
Micro-toroid:	2.6	1.25×10^5

Spillane et al, PRA 2005

Soda et al, Nature Materials 2005



Summary

- Photon nonlinearities

  EIT-based schemes

  Stark-shift based scheme

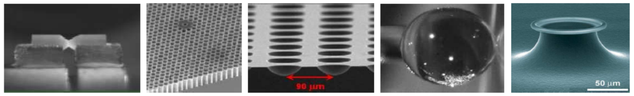
- Photonic Bose-Hubbard models

  Polaritons in coupled array of cavities

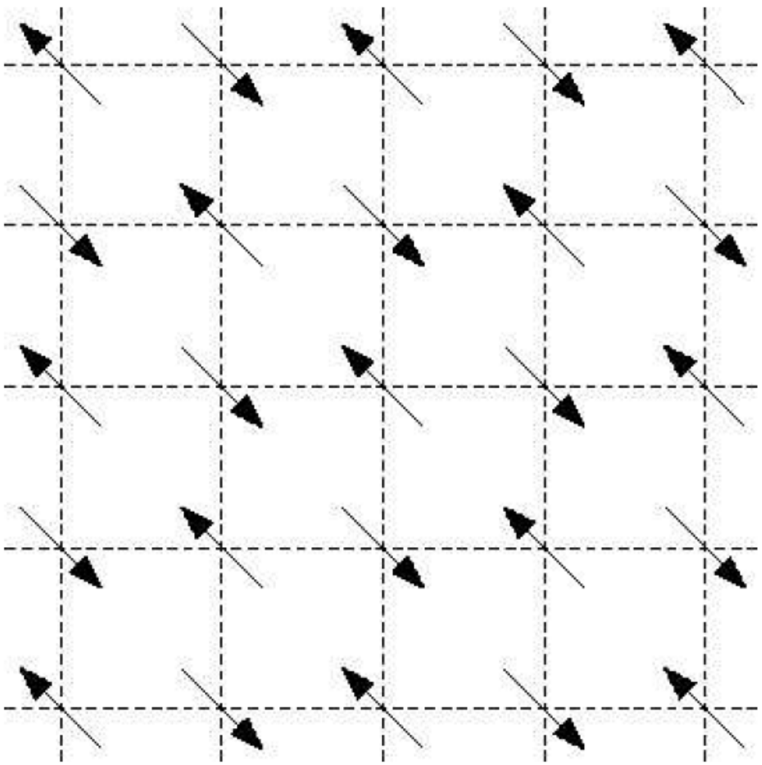
  The photonic limit

- Spin Chains

  Heisenberg model (XYZ)



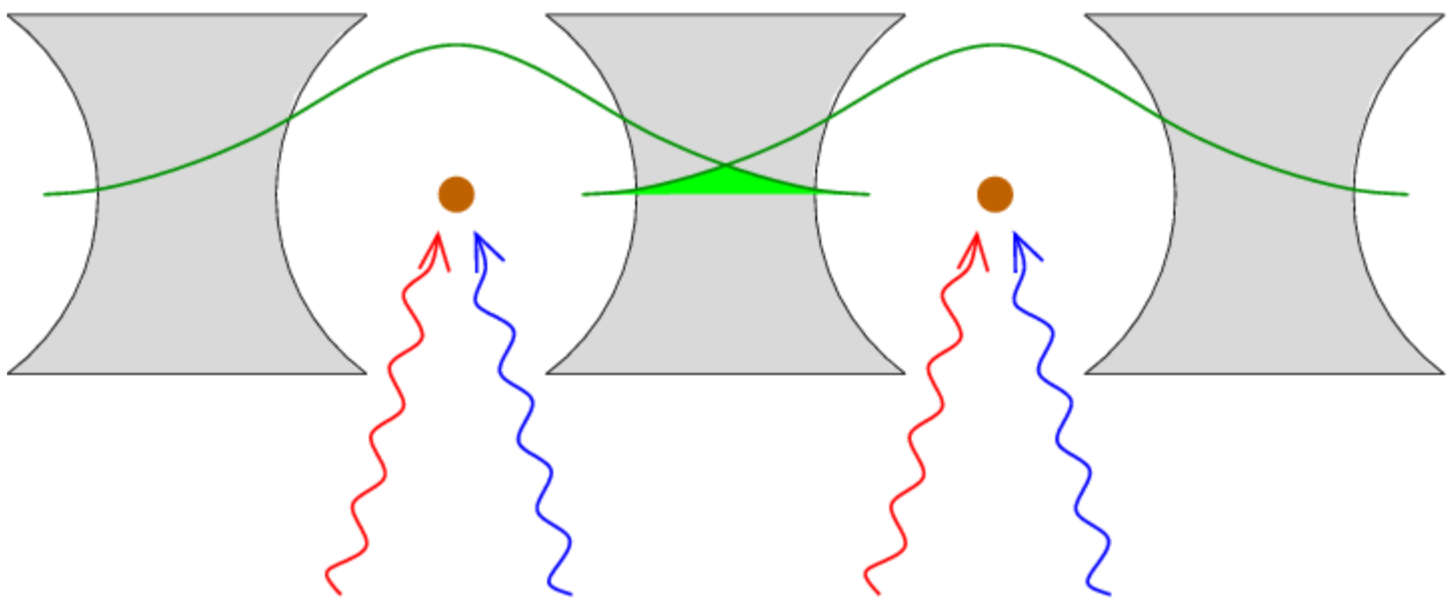
Spins Lattices



- Open questions in condensed-matter physics: high T_c superconductivity, frustration, etc…
- Applications in quantum information science: entanglement propagation, measurement-based quantum computation, etc…

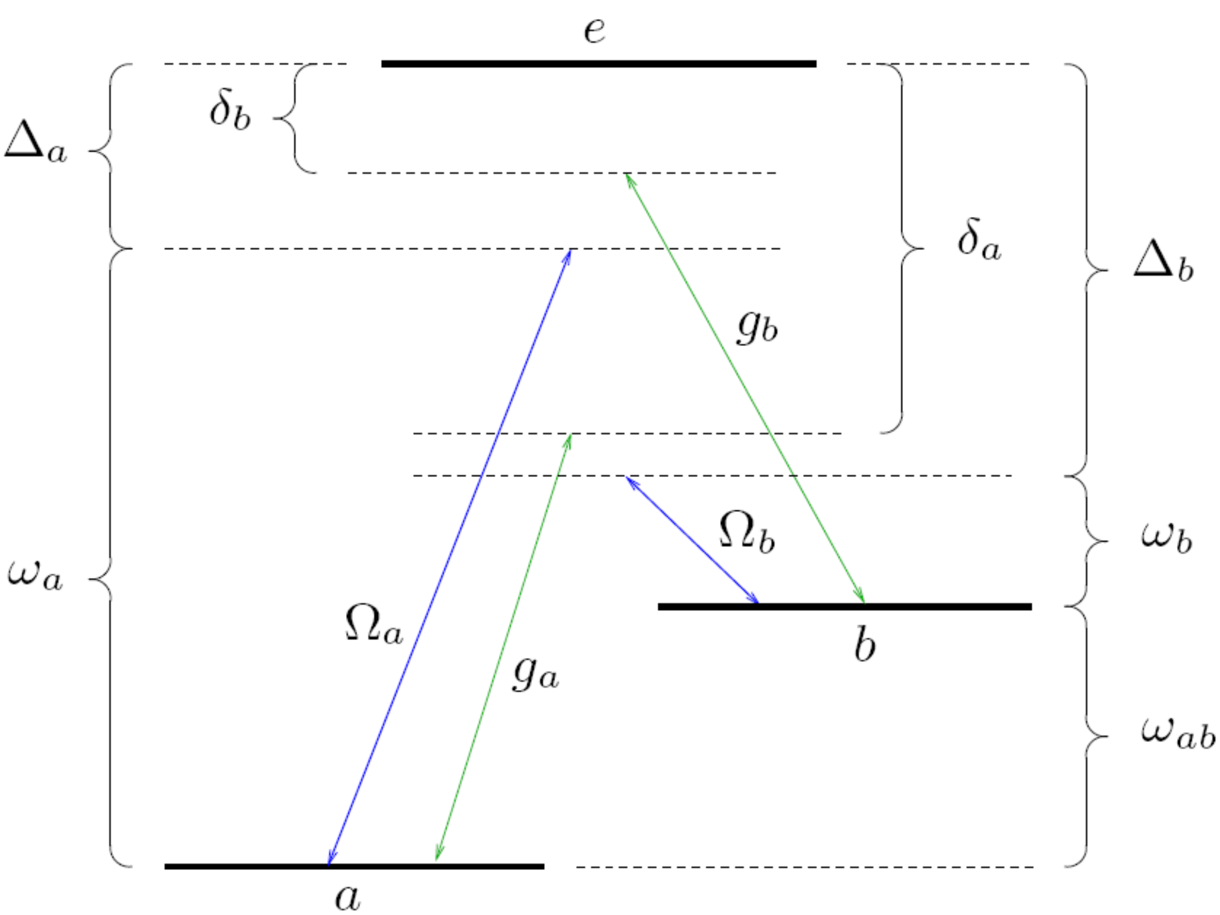
Spins Lattices: Heisenberg (XYZ) model

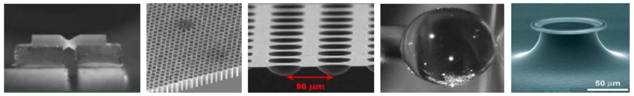
$$\mathcal{H} = \sum_j \left(J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y + J_z \sigma_j^z \sigma_{j+1}^z + B \sigma_j^z \right)$$



Spins Lattices: Heisenberg (XYZ) model

XX and YY interactions:





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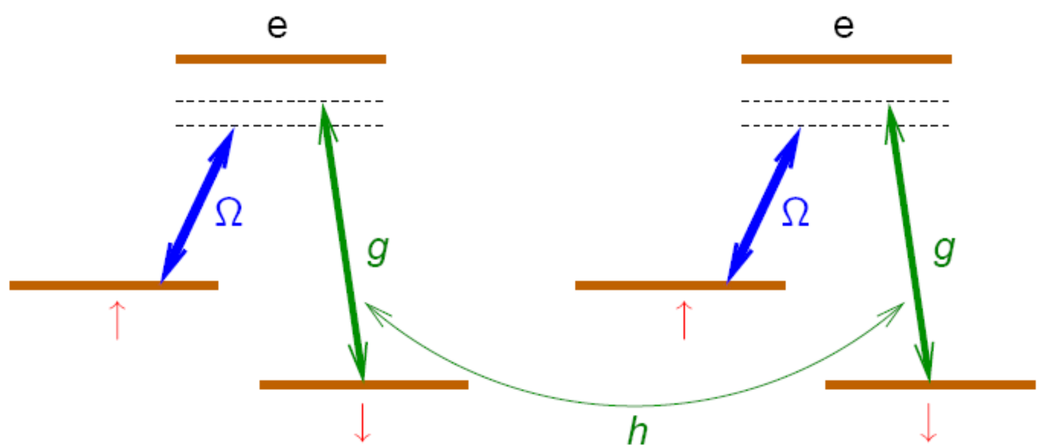
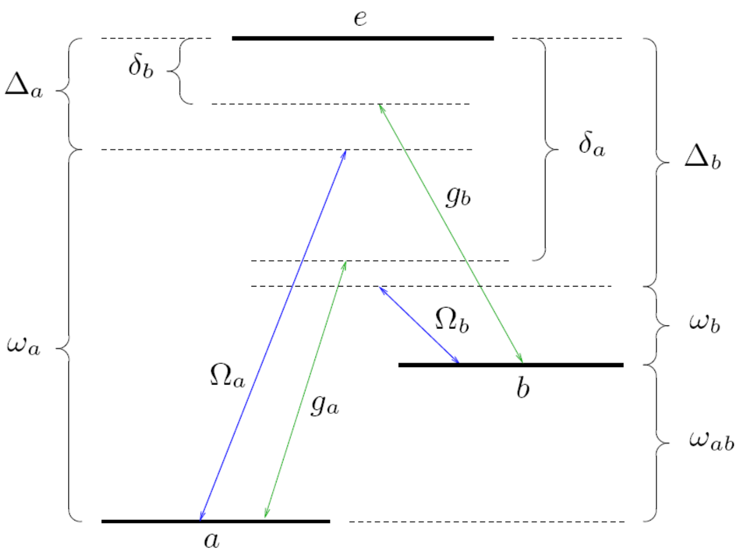
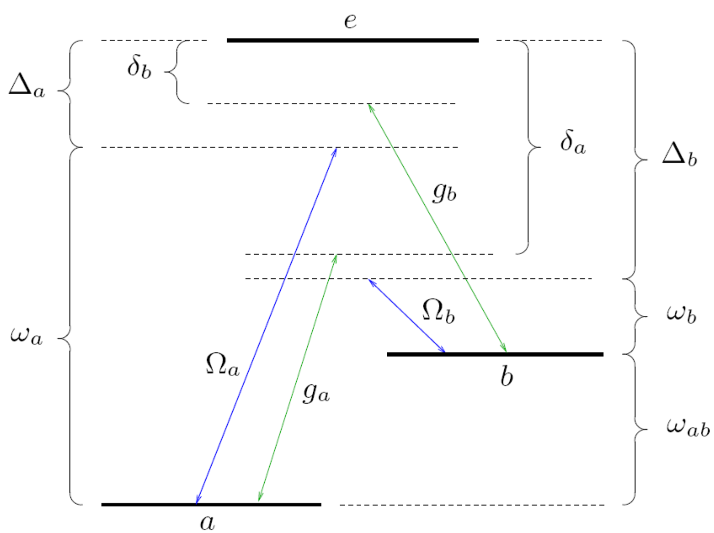
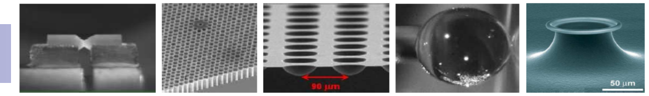


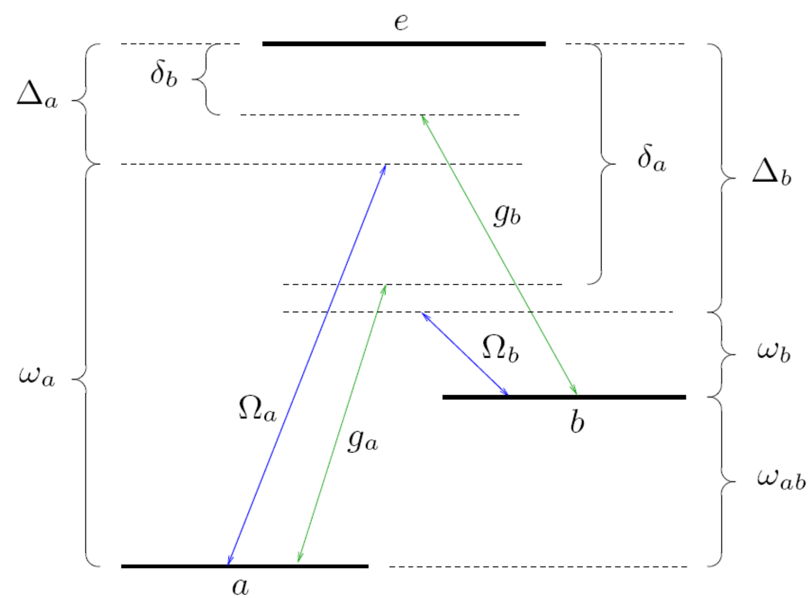






Spins Lattices: Heisenberg (XYZ) model

XX and YY interactions:

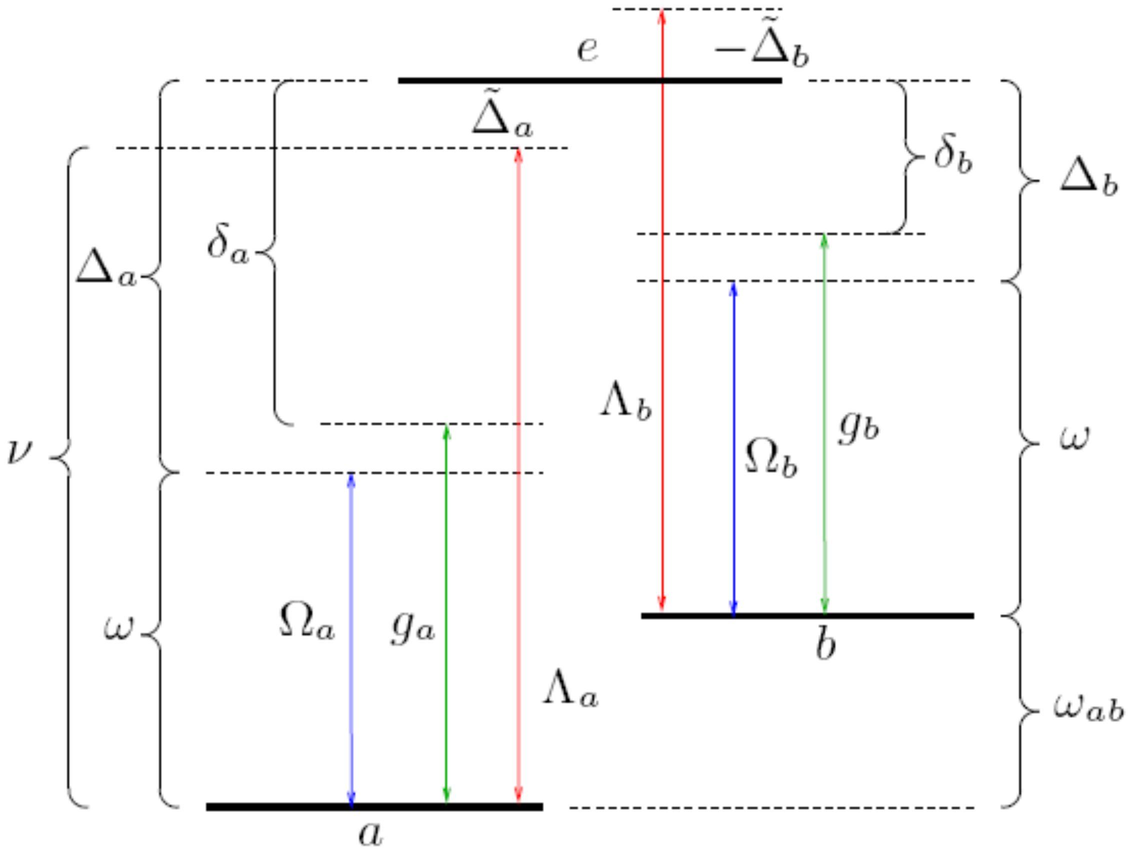


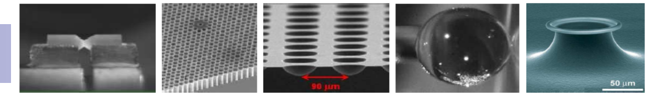
$$H_{XY} = \sum_j \left(\frac{J_1 + J_2}{2} \sigma_j^x \sigma_{j+1}^x + \frac{J_1 - J_2}{2} \sigma_j^y \sigma_{j+1}^y + B\sigma_z \right)$$

$$J_1 = \frac{\gamma_2}{4} \left(\frac{|\Omega_a|^2 g_b^2}{\Delta_a^2} + \frac{|\Omega_b|^2 g_a^2}{\Delta_b^2} \right) \quad J_2 = \frac{\gamma_2}{2} \frac{\Omega_a^\star \Omega_b g_a g_b}{\Delta_a \Delta_b}$$

Spins Lattices: Heisenberg (XYZ) model

ZZ interactions + magnetic field:

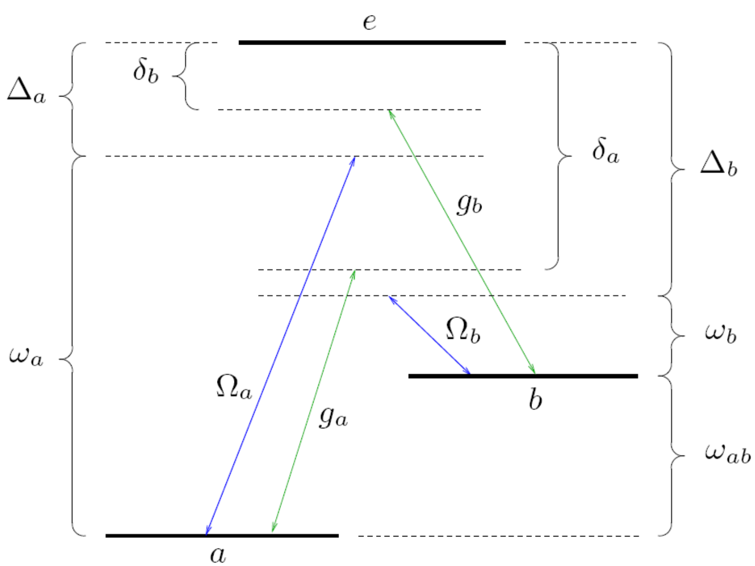




Spins Lattices: Heisenberg (XYZ) model

$$\mathcal{H} = \sum_j \left(J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y + J_z \sigma_j^z \sigma_{j+1}^z + B \sigma_j^z \right)$$

Suzuki-Trotter Decomposition:



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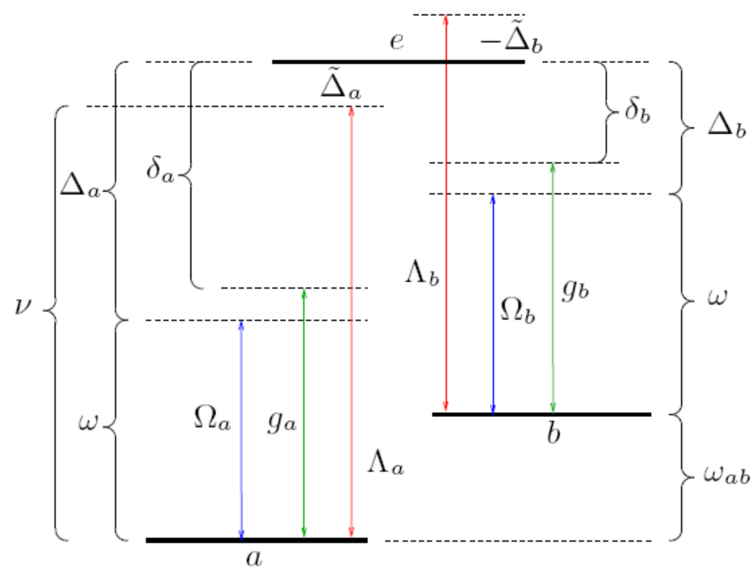
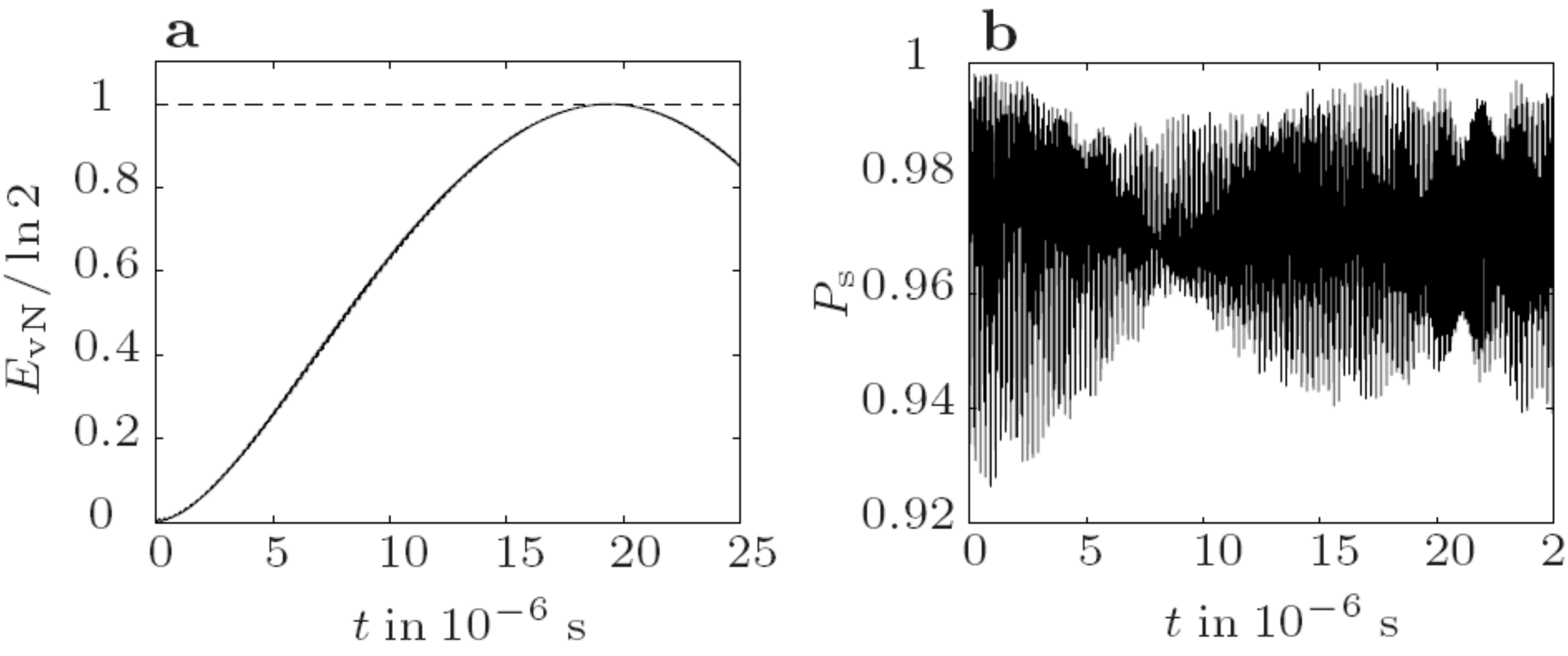
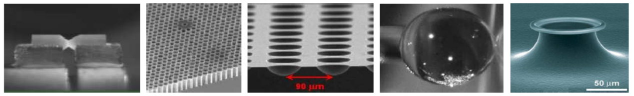


Cluster state generation

$$H_{\rm zz} = \sum_{j=1}^{N} \left(\tilde{B}\sigma_j^z + J_z \sigma_j^z \sigma_{j+1}^z \right)$$





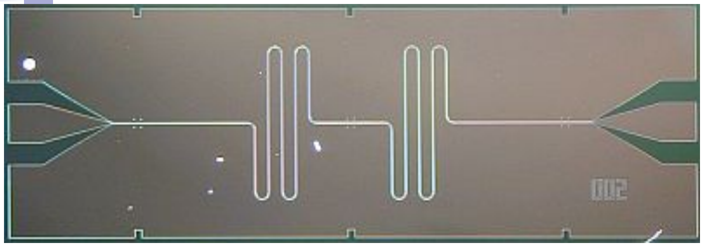
Spins Lattices: XYZ model

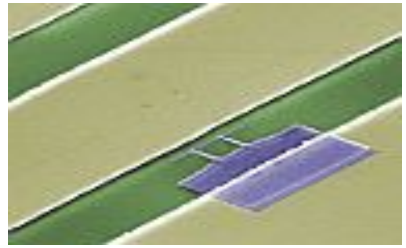
little occupation in "e" $\Rightarrow$ spontaneous emission suppressed

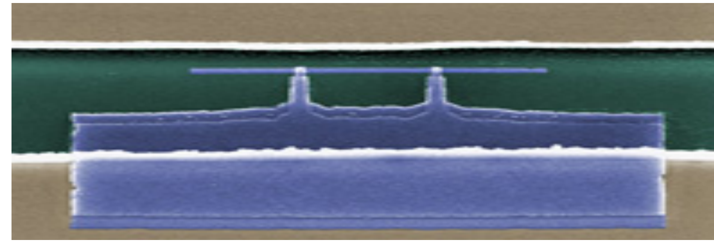
only virtual photons $\Rightarrow$ cavity decay suppressed

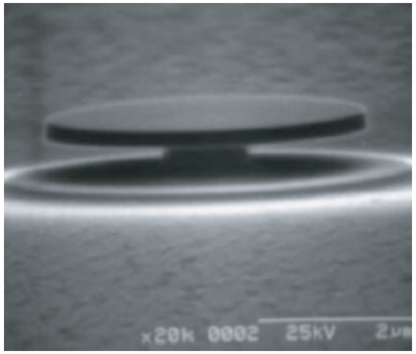
$$\kappa << h \qquad\qquad \gamma << g$$

$$\kappa\gamma << g^2$$



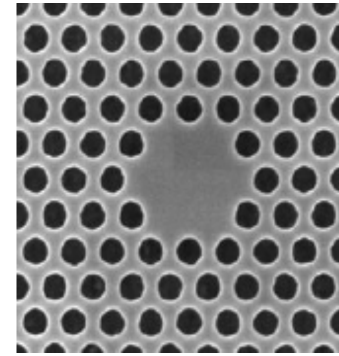


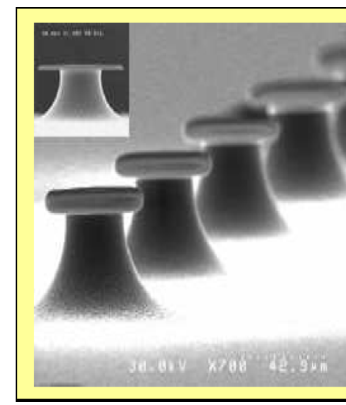


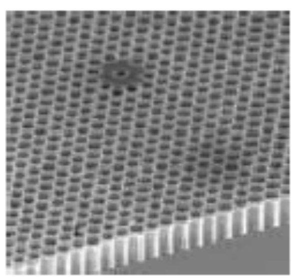


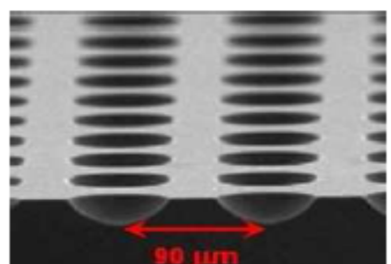
	$g^2/\gamma\kappa$		g/γ	
	real	pred.	real	pred
Fabry-Perot:	160	5×10^3	60	420
Photonic bc:	10	5.5×10^5	100	10^5
MCs @ Imperial:	40	?	50	?
Micro-toroid:	53	5×10^6	20	400

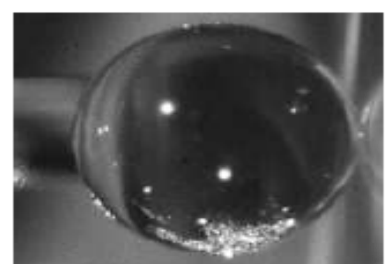
Spillane et al, PRA 2005 Soda et al, Nature Materials 2005

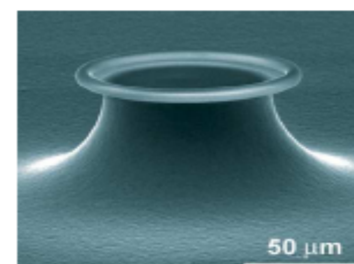


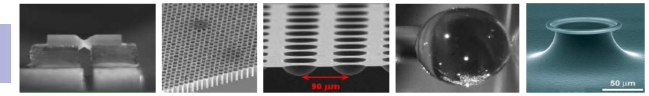












References

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  Hartmann, Plenio, arXiv:0704.2575

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- Bose Hubbard model: Hartmann, Brandão, Plenio, Nature Physics **2**, 849 (2006), quant-ph/0606097

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  Angelakis, Santos, Bose, quant-ph/0606159

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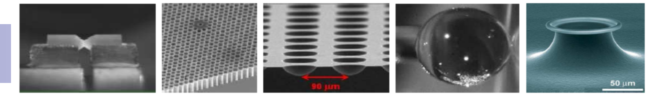

Thank you!

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