Physics 125b

Problem Set 5, Due Wednesday Feb. 28, 2018

Problem 1

In class we found that the Green's function satisfying

$$(\nabla^2 + k^2)G^0(\vec{r}) = \delta^2(\vec{r})$$

is useful in scattering. When we tried to get an explicit expression for $G^0(\vec{r})$ a divergent integral was encountered which we regulated by adding $i\epsilon$, ($\epsilon > 0$) to the denominator. Suppose we regulated it by adding $-i\epsilon$. Evaluate the Green's function

$$G^0(\vec{r}) = \lim_{\epsilon \to 0} \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{k^2 - q^2 - i\epsilon}$$

Comment on the difference between this Green's function and the one we found in class.

Problem 2

A particle of mass m is scattered by the central potential,

$$V(r) = -\frac{\hbar^2}{ma^2} \frac{1}{\cosh^2(r/a)}, \tag{1}$$

where a is a constant. Given the equation

$$\frac{d^2y}{dx^2} + k^2 y + \frac{2}{\cosh^2 x} y = 0 \tag{2}$$

has the solutions $y = e^{\pm ikx}(\tanh x \mp ik)$, calculate the s-wave contribution to the total scattering cross section at energy E

Problem 3

Suppose the scattering amplitude for a certain reaction is given by

$$f(\theta) = \frac{1}{k}\left(\frac{k\Gamma}{k_0 - k - ik\Gamma} + 3e^{2i\beta k^3}\sin(2\beta k^3)\cos\theta\right), \tag{3}$$

where Γ, k_0, and β are constants characteristic of the potential which produces the scattering. Of course $k_0 = \sqrt{(2mE/\hbar^2)}$ is the de Broglie wave number.

- Which partial waves are active? (i.e. what values of l?). Justify your answer.
- Calculate the values of the non-zero partial wave amplitudes a_l and the corresponding phase shift δ_l.
- Calculate the total cross section by integrating the differential cross section.
- Compare with the cross section predicted by the optimal theorem:

$$\sigma = \frac{4\pi}{k}\mathrm{Im}f(\theta = 0). \tag{4}$$