Physics 125a

Problem Set 4, Due Wed. Nov 9, 2015

Problem 1

A particle of mass m moves in one dimension under the influence of a harmonic oscillator potential

$$H = \frac{P^2}{2m} + \frac{m\omega^2}{2} X^2 \tag{1}$$

The particle is in the ground state. Suddenly, at time $t = 0$ the value of ω increases by a factor of 4. The sudden change in ω doesn't change the sate of the system. But now the particle is not in the ground state of the new Hamiltonian

$$H_{\text{new}} = \frac{P^2}{2m} + \frac{m(4\omega)^2}{2} X^2 \tag{2}$$

The new Hamiltonian H_{new} is still of the Harmonic oscillator form so it has eigensates $|n\rangle_{\text{new}}$ with energy $E_{\text{new}} = 4\hbar\omega(n + 1/2)$.

(a) Let R be the probability that just after changing the value of ω the particle is in eigenstate $|n+2\rangle_{\text{new}}$ divided by the probability of it being in eigenstate $|n\rangle_{\text{new}}$. Calculate R.

(b) Just after the change in ω what is the probability of the particle being in the eigenstate $|1\rangle_{\text{new}}$.

Problem 2

Two identical non-interacting particles of mass m are in a one-dimensional box of length L. Measurement of the energy of the system yields the value $E = \hbar^2\pi^2/(mL^2)$. Write down the possible state vectors for the system. Repeat this for $E = 5\hbar^2\pi^2/(2mL^2)$. You are not told if the particles are fermions or bosons.

Problem 3

Imagine a situation where there are three particles and only three one particle states $|a\rangle$, $|b\rangle$ and $|c\rangle$ available to them. Show that the total number of allowed (distinct) states for the system is

(a) 27 if the particles are not identical

(b) 10 if the particles are bosons

(c) 1 if the particles are fermions.

Problem 4

A non relativistic particle of mass m and charge q moves in the static magnetic field that corresponds to the vector potential $\mathbf{A} = (B/2)(-y\hat{\mathbf{x}} + x\hat{\mathbf{y}})$

(a) Show that the magnetic field is $\mathbf{B} = B\hat{\mathbf{z}}$

(b) Prove that a classical particle in this potential will move in circles at the angular frequency $\omega_0 = qB/(mc)$.

(c) Show that the Hamiltonian for this system can be written as the sum of the Hamiltonial for a two dimensional Harmonic oscillator with frequency $\omega_0/2$ and an additional term. What observable does the additional term correspond to?

(d) Find the allowed energy levels of the system.