# Physics 125b

## Problem Set 4, Due Wednesday Feb. 21, 2018

### Problem 1

Suppose the nuclear charge is confined to the surface of the atomic nucleus so that $V(r) = -Ze^2/r_0$ for $r < r_0$ and $V(r) = -Ze^2/r$ for $r > r_0$. Use the trial wave function $\psi \propto e^{-r/a}$ to calculate the variational ground sate energy $E_{\text{var}}$ (in electron volts) for an electron in this potential. Assume $r_0 = 10^{-12}$ cm, $Z = 6$. Compare this variational estimate with the ground state energy $E_0$ for the Coulomb problem where all the charge is at the origin. In particular calculate, $(E_{\text{var}} - E_0)/E_0$.

*Hint*: We expect the the value of $a$ to be very close to what you would get in the Coulomb problem. Use that fact and expand in the difference between the variational $a$ that minimizes the energy and the Coulomb value and then approximately solve for that difference.

### Problem 2

A typical cross section at LHC energies is a picobarn. Suppose the cross section for scattering is 1 picobarn=$10^{-12}$ barn = $10^{-40}$ meters$^2$. The beam of incident particles is moving very near the speed of light. What beam number density gives a rate of scattered events that is 100 per day?

### Problem 3

Use the Born approximation to derive and expression for the cross section for a particle of mass $m$ scattering off the potential $V(r) = -V_0\theta(r_0 - r)$. What does it become in the limit $kr_0 \to 0$. Recall that the theta function is defined by $\theta(z) = 1$ for $z > 0$ and $\theta(z) = 0$ for $z < 0$.

## Problem 4

Consider two spin 1/2 particles $a$ and $b$. Particle $a$ is the beam particle and particle $b$ is the target particle. Particle $b$ is very heavy so we treat is as at rest at the origin $r = 0$.

**(a)** Derive an expression for the differential cross section in the Born approximation when spin effects are included. Take the initial state to be $|\vec{p}_i\rangle \otimes |\chi_i\rangle$ and the final state to be $|\vec{p}_f\rangle \otimes |\chi_f\rangle$ . Here $\vec{p}_{i,f}$ are the initial and final momenta of particle $a$ and $\chi_{i,f}$ is the spin state of particles $a$ and $b$ before and after the scattering. Assume the potential is a product of a terms that depend on spin and position.

**(b)** Suppose the beam and target particles initially have opposite spins along the beam direction which we take to be along the $z$-axis. More explicitly the initial spins are eigenstates of $S_a^z$ and $S_b^z$ with eigenvalues $\hbar/2$ and $-\hbar/2$. Calculate in the Born approximation the differential cross section using the potential

$$V(r) = \vec{S}_a \cdot \vec{S}_b f(r),$$

where $r$ is the radial coordinate and $\vec{S}_{a,b}$ are the spin operators for particles $a$ and $b$. You can express your answer in terms of the Fourier transform of $f(r)$. Remember to sum over the possible final states of the beam and target particles since the spins of the beam and target particle are not being measured after the scattering.