Physics 125a

Problem Set 3, Due Wed. Nov 26, 2016

Problem 1

Find $\langle X\rangle$, $\langle P\rangle$, ΔP and ΔX for a particle of mass m in a one dimensional harmonic oscillator, $V(X) = (1/2)m\omega^2 X^2$, in energy eigenstate $|n\rangle$. (In this class our particles are always non relativistic with the standard kinetic term unless explicitly stated otherwise).

Problem 2

This is a continuation of problem 1. Suppose at time $t = 0$ the particle starts out in the state $|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$.

(a) Calculate $\langle\psi(0)|X|\psi(0)\rangle$ and $\langle\psi(0)|P|\psi(0)\rangle$.

(b) Find $|\psi(t)\rangle$ and use it to compute $\langle\psi(t)|X|\psi(t)\rangle$ and $\langle\psi(t)|P|\psi(t)\rangle$.

Problem 3

The Nobel Prize for physics in 2005 was awarded in part to Roy Glauber for his work on the properties of so called coherent states of the harmonic oscillator, particularly as they apply to the quantum theory of optics. In this problem and the next you will work out a few of the basic properties of coherent states (alas more than forty years too late for it to be Nobel Prize winning material).

A coherent state $|\lambda\rangle$ of the one dimensional harmonic oscillator is defined to be an eigenstate of the destruction operator a

$$a|\lambda\rangle = \lambda|\lambda\rangle$$

(Note that since a is not Hermitian, λ will in general be a complex number and the coherent states will not form an orthonormal basis.) The ground state $|0\rangle$ is a coherent state with $\lambda = 0$.

(a) Show that

$$|\lambda\rangle = e^{-\frac{|\lambda|^2}{2}} e^{\lambda a^\dagger} |0\rangle$$

is a properly normalized coherent state. (Hint: You might try first writing the second exponential as a sum of powers of $a^\dagger$ and then express $|\lambda\rangle$ as a sum over the energy eigenstates $|n\rangle$)

(b) Let $f(n) = \langle n|\lambda\rangle$. Show that the probability of the coherent state having energy corresponding to the level n of the Harmonic oscillator (sometimes we just say of being in the energy eigenstate $|n\rangle$) is

$$|f(n)|^2 = \frac{\nu^n}{n!} e^{-\nu}$$

with ν a positive real number. Give ν in terms of λ.

Problem 4

This is a continuation of problem 3 above.

(c) Show that the coherent states $|\lambda\rangle$ minimize the uncertainty relation product $\Delta X \Delta P$.

(d) Using the Hamiltonian for the (simple) Harmonic oscillator $H = \hbar\omega(a^\dagger a + 1/2)$ show that after a time t the coherent state $|\lambda\rangle$ evolves into another coherent state $|\lambda'\rangle$. Give λ' in terms of λ ω and t and indicate the overall phase of the new coherent state.