Physics 125b

Problem Set 2, Due Wed. Jan 24, 2018

Problem 1

Two identical spin 1/2 fermions of mass m move in a spherically symmetric harmonic oscillator potential,

$$H^{(0)} = \frac{(\vec{p}_1^2 + \vec{p}_2^2)}{2m} + \frac{m\omega^2}{2}(\vec{x}_1^2 + \vec{x}_2^2)$$

with an additional small spin dependent term

$$H^{(1)} = \epsilon(S_1^z z_2 + S_2^z z_1).$$

Here the subscripts label the particle type (1 or 2). The position vector $\vec{x}$ has cartesian coordinates (x, y, z). The spin operators along the $\hat{z}$ direction are $S_{1,2}^z$.

Does this perturbation break rotational invariance? Argue that to first order in perturbation theory the shift in the energy levels caused by the perturbation Hamiltonian $H^{(1)}$ vanishes.

Problem 2

This is a continuation of problem 2. Find the the energy of the ground state at second order in perturbation theory.

Problem 3

A particle of mass m moves in one dimension in the harmonic oscillator potential

$$H^{(0)} = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2. \tag{1}$$

At time $t = 0$ the perturbation

$$H^{(1)} = \eta\frac{P}{m}$$

is added and it is turned off at time $t = T$. Suppose at time $t = 0$ the particle is in the eigenstate of $H^{(0)}$ with energy $E_n = \hbar\omega(n + 1/2)$. What is the average energy loss or gain during the interval of time between time $t = 0$ and T? Use first order time dependent perturbation theory.