Physics 125a

Problem Set 2, Due Wed. Oct. 12, 2016

Problem 1

$|K^0\rangle$ and $|\bar{K^0}\rangle$ are two orthogonal states with unit norm that correspond to the particles K^0 and $\bar{K^0}$. These particles decay into other particles so if we approximate their physics as a two state system (leaving out the degrees of freedom corresponding to their positions, momentum and the degrees of freedom associated with their decay products) probability is not conserved. We can do this by introducing a Hamiltonian that is not Hermetian. Of course the real world Hamiltonian is Hermetian, we are only getting something not Hermetian because we made an approximation that left out some of the physics. This weird non Hermetian Hamiltonian is sometimes called an effective Hamiltonian for that reason. Suppose the effective 2×2 Hamiltonian matrx can be written as

$$H = M - i\frac{\Gamma}{2} \tag{1}$$

where M and Γ are Hermetian 2×2 matrices that have equal diagonal elements, $M_{11} = M_{22} = M$ and $\Gamma_{11} = \Gamma_{22} = \Gamma$.

(a) Suppose the off diagonal elements M_{12} and Γ_{12} are zero. If at time t=0 the system is in the state $|K^0\rangle$ what state is it at time $t = T$. To correspond the the physics described above what sign do we want to take for the real number Γ. Now suppose we have a collection of $N >> 1$ K^0 particles at time $t = 0$. How many are left at the time $t = T$. In the real world $\hbar/\Gamma \sim 10^{-10}$sec.

(b) Next suppose the off diagonal elements M_{12} and Γ_{12} are real what are the eigenstates and eigenvalues of the effective Hamiltonian expressed in terms of M, Γ, M_{12} and Γ_{12}.

(c) Assume as in part (b) that M_{12} and Γ_{12} are real. If we start at $t = 0$ in the state $|K^0\rangle$ what is the probability at a later time $t = T$ that the system is in the state $|\bar{K^0}\rangle$.

Problem 2

Suppose we have three compatable observables Ω, Λ and Γ. They have a basis of simultaneous eigenalues which we take to be orthornormal. In the usual notation we label the basis using the eigenvalues as $|\omega_i, \lambda_j, \gamma_k\rangle$. Suppose we prepare an ensemble of systems all in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}|\omega_1, \lambda_1, \gamma_3\rangle - \frac{1}{\sqrt{6}}|\omega_1, \lambda_2, \gamma_4\rangle + \frac{i}{\sqrt{2}}|\omega_2, \lambda_1, \gamma_4\rangle \tag{2}$$

(a) What is the norm of this state. Suppose we first measure Ω, next measure Λ and finally measure Γ. What is the probability that you get ω_1, λ_2 and γ_4. What state are you after each step in the measurement process.

(b) Repeat this for the other 5 orders of making the measurements of these three observables finding the state at the different stages of the measurement process and the probability of the final result.

Problem 3

Given the linear operators A, B and C. show that,

(a) $[A, BC] = [A, B]C + B[A, C]$ and $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$
.

(b) If $[A, B] = c$, where c is a complex number (the identity operator is implicit) use induction to show that $[A, B^n] = ncB^{n-1}$

Problem4

This is a continuation of Problem 3

(c) Let X and P be the position and momentum operators defined in class, obeying the usual commutation relation $[X, P] = i\hbar$. Evaluate the commutator $[X, e^{iPa}]$, where a is a constant.

(d) Show that that $e^{iPa}|x'\rangle$, where $X|x'\rangle = x'|x'\rangle$, is an eigenstate of the position operator X. What is the corresponding eigenvalue?