Physics 125b

Problem Set 1, Due Wed. Jan 25, 2017

Problem 1

Derive the Clebsh-Gordon coefficients for, $\frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$

Problem 2

Derive the Clebsh-Gordon coefficients for, $1 \otimes 1 = 2 \oplus 1 \oplus 0$

Problem 3

Consider a hydrogen atom with an additional angular momentum dependence in the Hamiltonian

$$H = \frac{\vec{P}^2}{2m_e} - \frac{e^2}{r} - \epsilon \frac{e^2}{r} \left(\frac{L_z^2}{\hbar^2 + \vec{L}^2} \right)$$

where L_z is the z-component of the angular momentum operator $\vec{L}$ and ϵ is a dimensionless positive constant

(a) Calculate $[L_x, H]$, $[L_y, H]$ and $[L_z, H]$. Is the Hamiltonian rotationally invariant? If not explain why you can still use the $|n, l, m\rangle$ basis for energy eigenstates of H.

(b) What are the energy levels of this system?

(c) What are the ground state(s) and first excited state(s) for $\epsilon = 3$?

Problem 4

Rather than parametrize rotation operators by a rotation about an axis we can use the parametrization,

$$U[R(\alpha, \beta, \gamma)] = e^{-i\alpha J_z/\hbar} e^{-i\beta J_y/\hbar} e^{-i\gamma J_z/\hbar} \tag{1}$$

(a) Construct the 3×3 matrix $D^{(1)}[R(\alpha, \beta, \gamma)] = \langle 1, m'|U[R(\alpha, \beta, \gamma)]|1, m\rangle$ as a product of three 3×3 matrices. Recall in this case the magnetic quantum numbers, m and m' go over the values $1, 0, -1$.

Suppose $|\psi\rangle = U[R(\alpha, \beta, \gamma)]|1, 1\rangle$.

(b) Show that $\langle\psi|J_x|\psi\rangle = \hbar\text{sin}\beta\text{cos}\alpha$, $\langle\psi|J_y|\psi\rangle = \hbar\text{sin}\beta\text{sin}\alpha$, $\langle\psi|J_z|\psi\rangle = \hbar\text{cos}\beta$.

(c) Show that there is no value of the angles α, β and γ for which $|\psi\rangle = |1, 0\rangle$.