Physics 125b

Problem Set 2, Due Wed. Feb 1, 2017

Problem 1

We want to solve for the first order shift in the energy levels for a particle moving in one dimension under the influence of the Hamiltonian, $H = H^{(0)} + H^{(1)}$ where $H^{(0)}$ is the harmonic oscillator Hamiltonian and

$$H^{(0)} = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 \tag{1}$$

and the perturbing Hamiltonian is the periodic potential,

$$H^{(1)} = \epsilon \cos(\mathrm{kX}) \tag{2}$$

for a small constant ϵ and some constant k. Solve for the shift in the ground state energy using the creation and annihilation operators, not the explicit wave functions.

Hint: To do this you need to write the cosine as a sum of two exponentials and then use a special case of the Baker Hausdorff formula. Namely if A and B are two operators and their commutator $[A, B]$ is equal to a complex number c times the identity operator then the Baker Hausdorff formula becomes: $e^A e^B = e^{c/2} e^{A+B}$.

Problem 2

Consider a spin 1 particle (with no spatial degrees of freedom). The Hamiltonian is

$$H = AS_Z^2 + B(S_x^2 - S_y^2) \tag{3}$$

where S_i are the 3×3 spin matrices and A and B are real positive constants that satisfy $A \gg B$. Treating the B term as a perturbation find the eigenstates of $H^{(0)} = AS_z^2$ that are suitable for doing perturbation theory in B. Calculate the energy shifts to first order in B.

Problem 3

A particle of mass m moves in the Harmonic oscillator potential $H^{(0)} = P^2/(2m)+(1/2)m\omega^2X^2$. Suppose we add a perturbation that in the classical limit slows the particle down when it is going fast, $H^{(1)} = \gamma|P/m|$. What sign of γ does this correspond to. This looks pretty strange if you work in coordinate space where the momentum is proportional to a derivative. The absolute value of a derivative, what's that? So don't do that. Instead work in momentum space where your wave functions are functions of p and solve for the shift in the ground state energy from this perturbation to linear order in the small constant γ.