Physics 125b – Midterm – Due Wednesday February 15, 2017

Instructions

You have up to three hours to do the exam once you start. You can use your notes, my notes on the web, your problem sets (and solutions) and the text Shankar. The exam is due Wednesday February 15 before 4:00 pm. You can drop it off in class on Wednesday or at the homework dropbox. Each problem is worth 10 points.

Problem 1

In class we discussed how the Clebsch-Gordan coefficients allow us to express eigenstates $|j,m\rangle$ of total angular momentum (*i.e.*, $\vec{J}^2$ and J_z where $\vec{J} = \vec{J}_1 + \vec{J}_2$) in terms of a basis or product states $|j_1, m_1\rangle|j_2, m_2\rangle$ which are eigenstates of the individual particle angular momenta (*i.e.*, $\vec{J}_i^2$ and J_{iz} where $i = 1, 2$ labels particle type). The states $|jm>$ and $|j_1, m_1\rangle|j_2, m_2\rangle$ form two bases for the same space and are related by a unitary transformation through the Clebsch-Gordan coefficients. The two basis have the same dimension since j goes from $|j_1 - j_2|$ to $|j_1 + j_2|$ and each value of j occurs once. The same can be done for three angular momentum. The states of total angular momentum $|j,m\rangle$ can be expressed in terms of the product states $|j_1, m_1\rangle|j_2, m_2\rangle|j_3, m_3\rangle$ where now the total angular momentum vector is $\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3$.

(a) For the case $j_1 = j_2 = 1$ and $j_3 = 1/2$ what are the allowed values of total angular momentum j and how many times does each of these j's occur. Verify that the two basis have the same dimension.

(b) For the case $j_1 = j_2 = 1$ and $j_3 = 1/2$ of part **(a)** use the Clebsh-Gordan coefficients $\langle 1, m_1; 1, m_2|2, m'\rangle$ and $\langle 2, m'; 1/2, m_3|5/2, m\rangle$ to express the six states with total angular momentum $j = 5/2$, (*i.e.*, $|5/2, m\rangle$), as a linear combinations of the states $|1, m_1\rangle|1, m_2\rangle|1/2, m_3\rangle$. You don't have to put in the explicit values for these Clebsch-Gordan coefficients.

Problem 2

A particle of mass m moves in three dimensions in the spherically symmetric (discontinuous) potential $V(r) = (1/2)m\omega^2r^2$ for $r < r_0$ and $V(r) = 0$ for $r > r_0$. As long as r_0 is large enough it is a reasonable approximation to take the ground state to be that of the harmonic oscillator Hamiltonian, $H_{\rm h.o} = p^2/(2m) + (1/2)m\omega^2r^2$. Use first order perturbation theory to derive an expression for the ground state energy taking the unperturbed Hamiltonian to be, $H^{(0)} = H_{\rm h.o}$. You can express your answer in terms of the integral, $I(a,n) = \int_a^\infty dx x^n e^{-x^2}$. Recall that the harmonic oscillator ground state wave function is $\psi_{0,0,0} = (m\omega/\pi\hbar)^{3/4}\exp(-m\omega r^2/(2\hbar))$ with energy $E_{0,0,0} = (3/2)\hbar\omega$. What is the dimensionless quantity that has to be small for first order perturbation theory to be a good approximation?

Problem 3

Consider a case where the Hamiltonian for a particle of mass m is the Coulomb potential with a periodic time dependence in the particle charge, $H = \vec{p}^2/(2m) - q(t)^2/r$, where $q(t) = e(1 + \epsilon\sin(\omega t))$. Treating ϵ as very small and taking for the unperturbed Hamiltonian, $H^{(0)} = \vec{p}^2/(2m) - e^2/r$, use time dependent perturbation theory to to find an expression for the probability that the particle transitions in the time period $t = [0, \Delta t]$ from the ground state of $H^{(0)}$ with principal quantum number $n = 1$ to each of the four excited states of $H^{(0)}$ with principal quantum number $n = 2$. Treat ϵ and Δt as very small and work to leading non-trival order in these constants. Express your answers in terms of: the fine structure constant $\alpha = e^2/(\hbar c)$, the speed of light c, Δt, ω, the Bohr radius a_0 and ϵ. Verify that the final expressions for the probabilities are dimensionless. Recall that the Coulomb wave functions are,

$$\psi_{1,0,0} = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}, \quad \psi_{2,0,0} = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} (2 - r/a_0)e^{-r/(2a_0)}, ...$$

Hint: $\int_0^\infty dr r^n e^{-r} = n!$