# Physics 125b – Midterm – Due Wed February 7, 2018

## Instructions

You have up to three hours to do the exam once you start. You can use your notes, the notes on the web, your problem sets (and solutions) and the text Shankar. Mathematica will not be useful and so is not permitted. The exam is due Wed February 7 before 4:00 pm. You can drop it off in class on Wed or at the drop-off box. Each problem is worth 10 points.

## Problem 1

A particle of mass $m$ moves in three dimensions in the spherically symmetric (discontinuous) potential $V(r) = (1/2)m\omega^2 r^2$ for $r < r_0$ and $V(r) = 0$ for $r > r_0$. As long as $r_0$ is large enough it is a reasonable approximation to take the ground state to be that of the harmonic oscillator Hamiltonian, $H_{\text{h.o}} = p^2/(2m) + (1/2)m\omega^2 r^2$. Use first order perturbation theory to derive an expression for the ground state energy taking the unperturbed Hamiltonian to be, $H^{(0)} = H_{\text{h.o}}$. You can express your answer in terms of the integral, $I(a,n) = \int_a^\infty dx x^n e^{-x^2}$. Recall that the harmonic oscillator ground state wave function is $\psi_{0,0,0} = (m\omega/\pi\hbar)^{3/4}\exp(-m\omega r^2/(2\hbar))$ with energy $E_{0,0,0} = (3/2)\hbar\omega$. What is the dimensionless quantity that has to be small for first order perturbation theory to be a good approximation?

## Problem 2

A spin 1/2 particle with spin angular momentum operators $S_x$, $S_y$ and $S_z$ is fixed at the origin. Its spin degrees of freedom completely characterize its Hilbert space. The Hamiltonian for t ¡ 0 is,

$$H = H^{(0)} = \mu S_z. \tag{1}$$

At time $t = 0$ the Hamiltonian suddenly changes to $H = H^{(0)} + H^{(1)}$, where

$$H^{(1)} = g S_x \tag{2}$$

and then at $t = T$ the Hamiltonian suddenly changes back to $H^{(0)}$. Assume the constants $\mu, g$ are positive.

- For $t < 0$, assume the particle is in the groundstate of $H^{(0)}$. Use first order time dependent perturbation theory to determine the probability of finding the particle in the excited state of $H^{(0)}$ at time $t = T$.
- Now assume the Hamiltonian was always given by $H = H^{(0)} + H^{(1)}$. Treating $H^{(1)}$ as a small perturbation find the leading non-zero shift in the ground state energy level caused by $H^{(1)}$ and compare that result with the exact ground state energy eigenvalue of the full Hamiltonian $H$

## Problem 3

Consider a case where the Hamiltonian for a particle of mass $m$ is the Coulomb potential with a periodic time dependence in the particle charge, $H = \vec{p}^2/(2m) - q(t)^2/r$, where $q(t) = e(1 + \epsilon\sin(\omega t))$. Treating $\epsilon$ as very small and taking for the unperturbed Hamiltonian, $H^{(0)} = \vec{p}^2/(2m) - e^2/r$, use time dependent perturbation theory to to find an expression for the probability that the particle transitions in the time period $t = [0, \Delta t]$ from the ground state of $H^{(0)}$ with principal quantum number $n = 1$ to each of the four excited states of $H^{(0)}$ with principal quantum number $n = 2$. Treat $\epsilon$ and $\Delta t$ as very small and work to leading non-trival order in these constants. Express your answers in terms of: the fine structure constant $\alpha = e^2/(\hbar c)$, the speed of light $c$, $\Delta t$, $\omega$, the Bohr radius $a_0$ and $\epsilon$. Verify that the final expressions for the probabilities are dimensionless. Recall that the Coulomb wave functions are,

$$\psi_{1,0,0} = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}, \quad \psi_{2,0,0} = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} (2 - r/a_0)e^{-r/(2a_0)}, \ldots$$

*Hint:* $\int_0^\infty dr r^n e^{-r} = n!$