Physics 125b – Final – Due Friday March 17, 2017

Instructions

You have up to four hours to do the exam once you start. You can use your notes, the notes on the web, your problem sets (and solutions) and the text Shankar. The exam is due Friday March 16 before 4:00 pm. Each problem is worth 10 points.

Problem 1

A spin 1/2 particle with spin angular momentum operators S_x, S_y and S_z is fixed at the origin. Its spin degrees of freedom completely characterize its Hilbert space. The Hamiltonian for $t < 0$ is,

$$H = H^{(0)} = \mu S_z.$$

At time $t = 0$ the Hamiltonian suddenly changes to $H = H^{(0)} + H^{(1)}$, where

$$H^{(1)} = gS_x$$

and then at $t = T$ the Hamiltonian suddenly changes back to $H^{(0)}$. Assume the constants μ and g are positive.

(a) For $t < 0$ assume the particle is in the ground state of $H^{(0)}$. Use first order time dependent perturbation theory to determine the probability of finding the particle in the excited state of $H^{(0)}$ at time $t = T$.

(b) Now assume the Hamiltonian was always given by $H = H^{(0)} + H^{(1)}$. Treating $H^{(1)}$ as a small perturbation find the leading non-zero shift in the ground state energy level caused by $H^{(1)}$ and compare that result with the exact ground state energy eigenvalue of the full Hamiltonian H.

Problem 2

Use the Born approximation to calculate the differential cross section,

$$\frac{d\sigma}{d\Omega}(\theta, \phi)$$

for scattering off the square well potential $V = -V_0$ for $|x| < L$, $|y| < L$ and $|z| < L$. Outside this square box the potential vanishes. You can express the cross section in terms of the components of $\vec{q} = \vec{k}_f - \vec{k}_i$, where $\vec{k}_i = k\hat{z}$, and then give the components of the wave vector transfer $\vec{q}$ in terms of the angles θ and ϕ.

Problem 3

For identical spin zero particles in non-relativistic quantum field theory the non-interacting Hamiltonian is

$$H = \int d^3x \phi^\dagger(\vec{x}) \left(\frac{-\hbar^2 \nabla^2}{2m} \right) \phi(\vec{x})$$

One and two particle states with definite momentum and energy are $|\vec{k}_1\rangle = a^\dagger(\vec{k}_1)|0\rangle$ and $|\vec{k}_2, \vec{k}_3\rangle = a^\dagger(\vec{k}_2)a^\dagger(\vec{k}_3)|0\rangle$ respectively. Consider the small interaction term

$$H_{\text{int}} = \mu \int d^3x \left(\phi^\dagger(\vec{x})\phi^\dagger(\vec{x})\phi(\vec{x}) + \phi(\vec{x})\phi(\vec{x})\phi^\dagger(\vec{x}) \right).$$

What is the dimension of the constant μ? Calculate the transition matrix element $\langle \vec{k}_2, \vec{k}_3 | H_{\text{int}} | \vec{k}_1 \rangle$.