Physics 125b – Final – Due Friday 16 March, 2018

Instructions

You have up to four hours to do the exam once you start. You can use your notes, my notes on the web, your problem sets (and solutions) and the text Shankar. Mathematica will not be useful and so is not permitted. The exam is due Friday 16 March, 2018 before 4:00 pm. Problem 1 and 2 are worth 10 points each. Problems 3 and 4 are worth 5 points each.

Problem 1

Use the Born approximation to calculate the differential cross section,

$$\frac{d\sigma}{d\Omega}(\theta, \phi) \tag{1}$$

for scattering off the square well potential $V = -V_0$ for $|x| < L$, $|y| < L$, $|z| < L$. Outside this square box the potential vanishes. You can express the cross section in terms of the components of $\vec{q} = \vec{k}_f - \vec{k}_i$, where $\vec{k}_i = k\hat{z}$, and then give the components of the wave vector transfer $\vec{q}$ in terms of the angles θ and ϕ.

Problem 2

Let $\psi(x) = A(a^2 - x^2)$ if $|x| \leq a$ and $\psi(x) = 0$, if $|x| \geq a$. Use the variational principle with trial wave function $\psi(x)$ to find an upper bound on the groundstate energy of a harmonic oscillator (with $V(x) = m\omega_0^2 x^2$). Treat a as a variational parameter. Compare your answer to the actual ground-state energy.

Problem 3

The density matrix of an spin-1/2 particle in the S_z basis is

$$\rho = \begin{bmatrix} 1/4 & n \\ n^* & p \end{bmatrix} \tag{2}$$

where the asterisk represents complex conjugation.

1. What value must p have? Why?
2. What value(s) can n take?
3. What value(s) must n have for the density matrix to represent a pure state?

Problem 4

Derive the equation of motion

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)], \tag{3}$$

using Schrondiger's equation and the most general form of the density operator (i.e. $\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$).