Physics 125a – Midterm – Due November 2, 2016

Instructions

You have up to three hours to do the exam once you start. You can use your notes, my notes on the web, your problem sets (and solutions) and the text Shankar. Mathematica will not be useful and so is not permitted. The exam is due Wednesday Nov 2 before 4:00 pm. Each problem is worth 8 points (you get 1 point for handing in the exam).

Problem 1

A quantum mechanical observable is represented by the Hermetian operator O. It has an orthonormal basis of eigenstates $|\omega\rangle$, where $O|\omega\rangle = \omega|\omega\rangle$. Suppose a system is prepared in a state $|\Psi\rangle$ that is a linerar combination of eigenstates of O with eigenvalues $\omega = 0, \pm 1$

$$|\Psi\rangle = \frac{\alpha|1\rangle + \beta|-1\rangle + \gamma|0\rangle}{|\alpha|^2 + \beta^2| + |\gamma|^2}$$

where α, β and γ are complex numbers.

(a) Describe the possible outcomes and their probabilities if an observer measures O and then measures it again and multiplies the results of those two measurements.

(b) Describe the possible outcomes and their probabilities if an observer measures the obserbable O^2 .

Note: In this problem give both the state of the system (after each measurement is made) for each value of the observable obtained from the measurement and the probability of obtaining that outcome.

Problem 2

A two state system has a Hilbert space of state vectors, $|\Psi(t)\rangle = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}$ and time dependent Hamiltonian $H = gt \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, where g is a constant and t is time. (This problem corresponds to a spin 1/2 particle in a magnetic field directed along the x axis that grows linearly with time.)

(a) Express Schroedingers equation for the time evolution of $|\Psi(t)\rangle$ as first order coupled differential equations for the components $\psi_1(t)$ and $\psi_2(t)$. Taking linear combinations of the two equations convert them into uncoupled differential equations. Solve them and express $\psi_1(t)$ and $\psi_2(t)$ in terms of their initial values $\psi_1(0)$ and $\psi_2(0)$.

(b) Derive an expression for the probability that a particle inititially in the state with $\psi_1 = 1$, $\psi_2 = 0$ is in the state with $\psi_1 = 0$, $\psi_2 = 1$ after a time t.

Problem 3

A non-relativistic particle of mass m moves along the z axis under the infuence of the gravitational potential $V = mgz$. Recall in class we argued that in the classical approximation (stationary phase for the path integral)

$$\langle z_2|U(t_2, t_1)|z_1\rangle \simeq \text{Exp}\left(i\frac{S[z(t)]}{\hbar}\right)$$

where $S[z(t)]$ is the action for the classical path that joins $z_1(t_1)$ with the point $z_2(t_2)$. Using this approximation show that

$$\langle z_2|U(t_2, 0)|0\rangle \simeq \text{Exp}\left(im\frac{Az_2^2 + Bz_2 + C}{\hbar}\right)$$

and express A, B and C in terms of t_2, and g.