Physics 125a – Final – Due Friday December 9, 2016

Instructions

You have up to three hours to do the exam once you start. You can use your notes, my notes on the web, your problem sets (and solutions) and the text Shankar. Mathematica will not be useful and so is not permitted. The exam is due Friday December 9 before 4:00 pm. You can drop it off at my office. Each problem is worth 10 points.

Problem 1

A particle of mass m is moving in three dimensions under the influence of the spherically symmetric potential $V(r) = \kappa r$, where $\kappa > 0$. Suppose you guess that the ground state ($l = 0$) has a radial wave function,

$$R_{1,0} = \sqrt{\frac{1}{2a^3}} \exp\left(-\frac{r}{2a}\right).$$

Let a_{min} denote the value of the constant a that minimizes the expectation value of the Hamiltonian in this guess for the ground state. Find an expression for a_{min} in terms of $\hbar$, κ and m. Explain physically why it decreases as κ increases. You may use: $\int_0^\infty z^p \exp[-z] = p!$. This guess for the ground state wave function actually does not fall off fast enough as $r \to \infty$. What is the correct asymptotic behavior for very large r?

Problem 2

Two identical particles of mass m move in one dimension in the harmonic oscillator potential

$$V = \frac{1}{2} m\omega^2 (x_1^2 + x_2^2)$$

where x_1 and x_2 are the position coordinates of the two particles. Suppose one of the particles is in the ground state $|n\rangle = |0\rangle$ and the other in the first excited state $|n\rangle = |1\rangle$. Derive and expression for the expectation value $\langle (x_1 - x_2)^2 \rangle$ in the case the particles are fermions and in the case the particles are bosons. We are assuming that the spin states of the two particles are the same so that they don't play a role in the calculation of this expectation value. Note that the expectation value is larger for fermions than bosons and this is sometimes called Fermi-repulsion. Use the creation and destruction operators $a_{1,2}^\dagger$ and $a_{1,2}$ for your computations not the explicit wave functions.

Problem 3

In this problem we focus only on spin degrees of freedom. (Note we are back in three spatial dimensions so there are three spin operators.) At time $t = 0$ two distinguishable spin 1/2 particles A and B are in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[e^{i\alpha} |A, \uparrow\rangle_z |B, \uparrow\rangle_z + |A, \downarrow\rangle_z |B, \downarrow\rangle_z \right]$$

Here we adopt the notation ($S_z^{(A)}$ is the spin operator for particle A along the z direction)

$$S_z^{(A)} |A, \uparrow\rangle_z = \frac{\hbar}{2} |A, \uparrow\rangle_z \quad \text{and} \quad S_z^{(A)} |A, \downarrow\rangle_z = -\frac{\hbar}{2} |A, \downarrow\rangle_z$$

and similarly for particle B. At time $t = 0$ the spin of particle B along the x axis is measured.
(a) What are the probabilities of the two possible outcomes $\pm\hbar/2$?
(b) After each outcome is measured what is the resulting (normalized state) when all the spins are expressed in the basis of states polarized along the z axis.
(c) Repeat parts (a) and (b) in the case that the particle A is absent and the state is

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[e^{i\alpha} |B, \uparrow\rangle_z + |B, \downarrow\rangle_z \right]$$