On Minimum Reversible Entanglement Generating Sets

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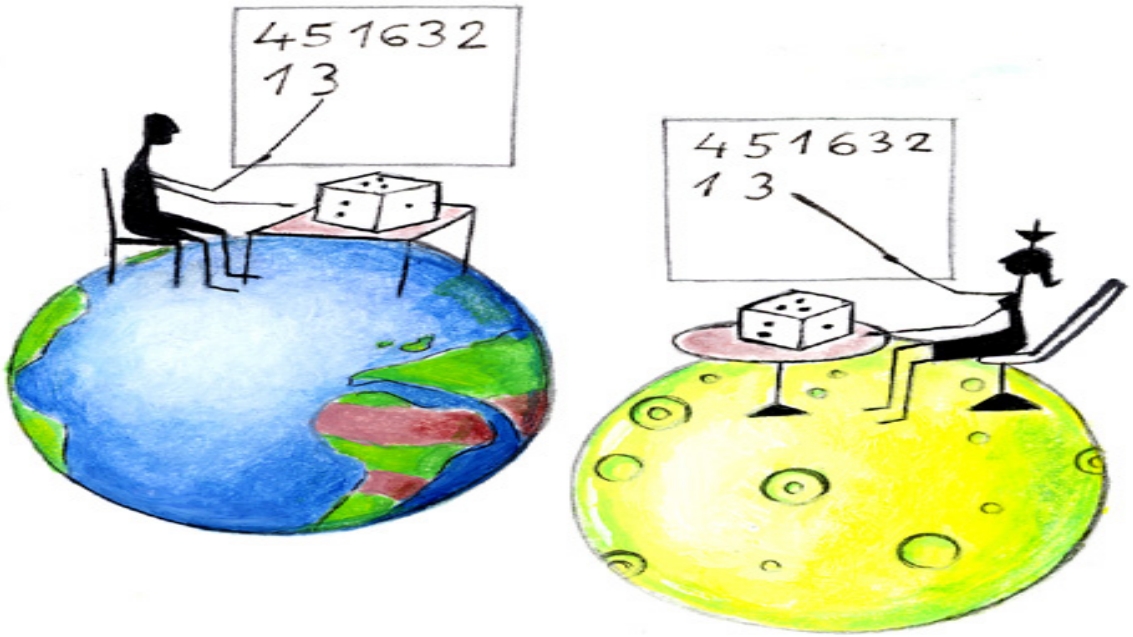
QIC

Quantum Entanglement



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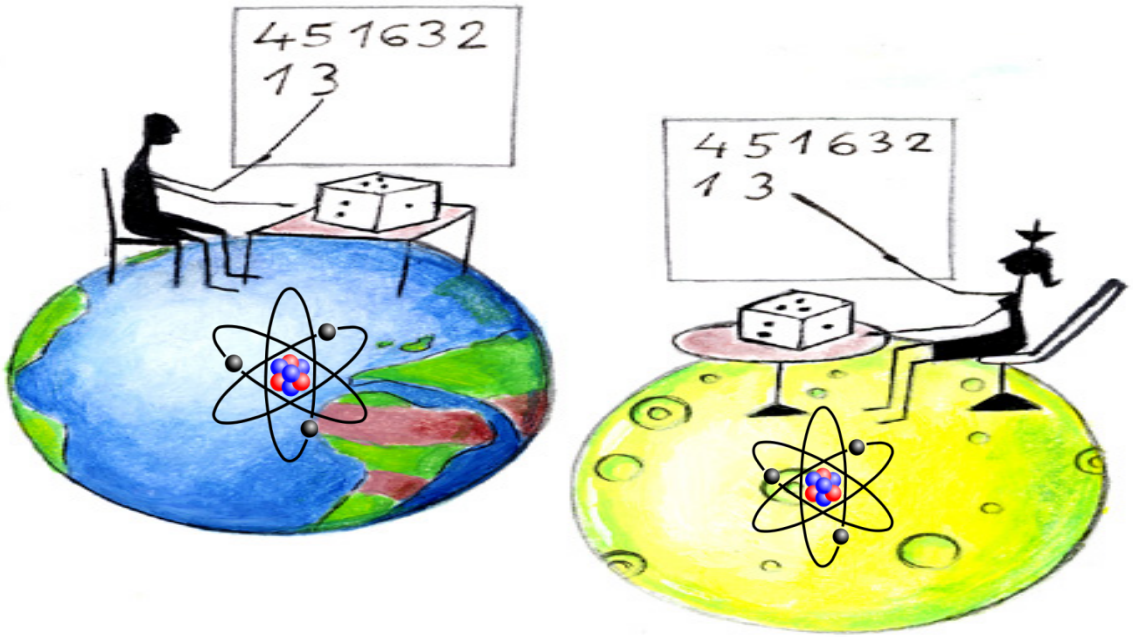

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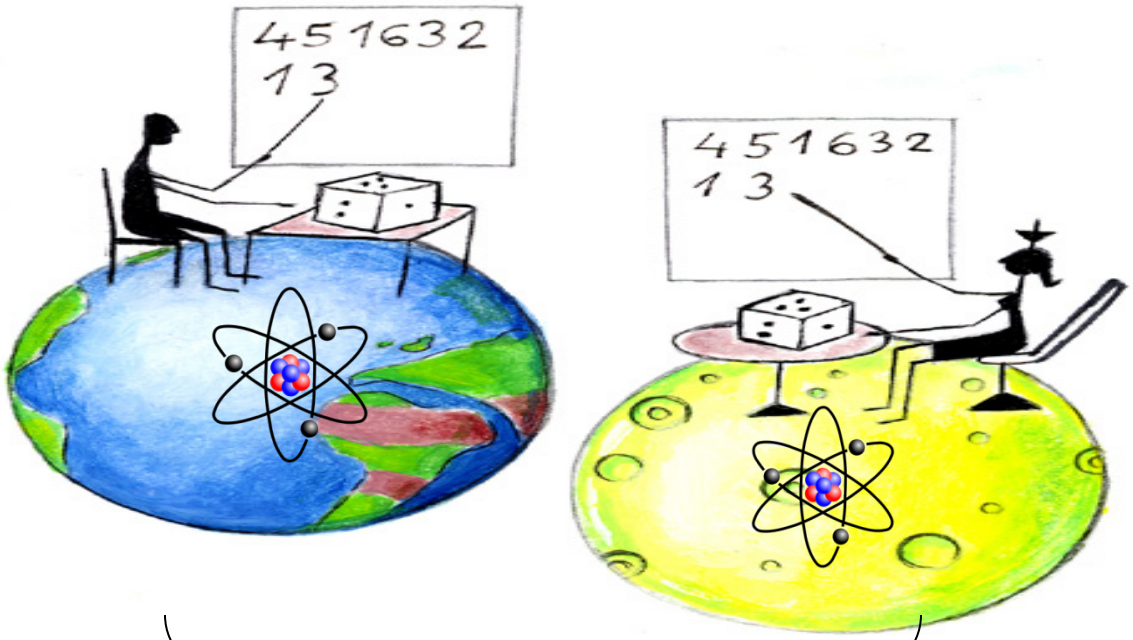
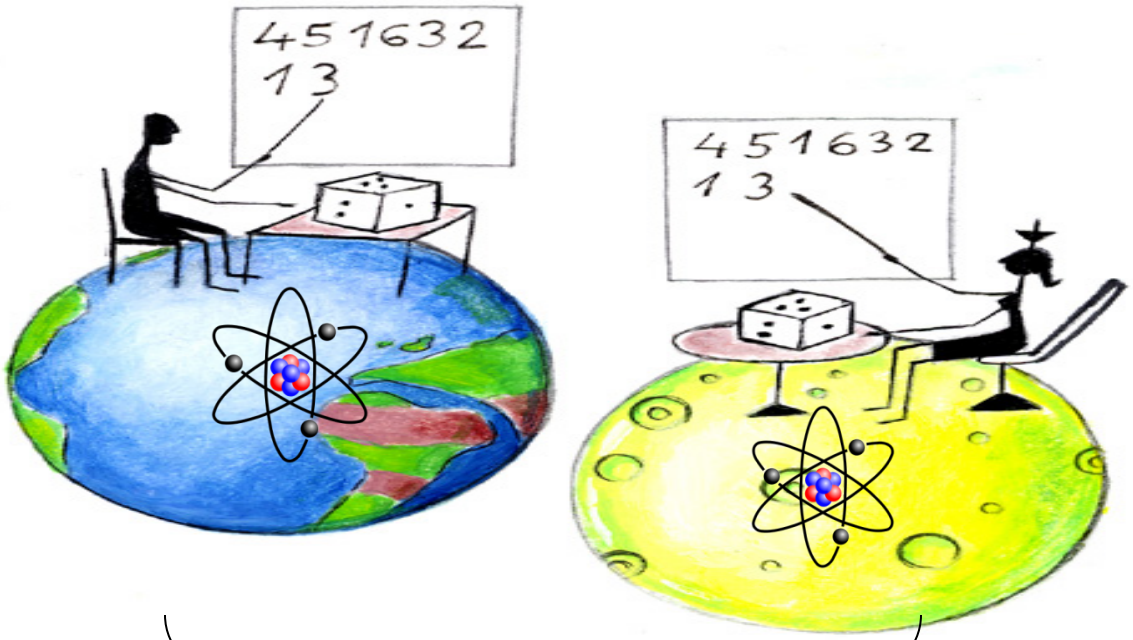


$$\sum_i p_i \rho_A^{\ i} \otimes \rho_B^i$$

Quantum Entanglement



$$\rho \neq \sum_i p_i \rho_A^{\,i} \otimes \rho_B^i$$

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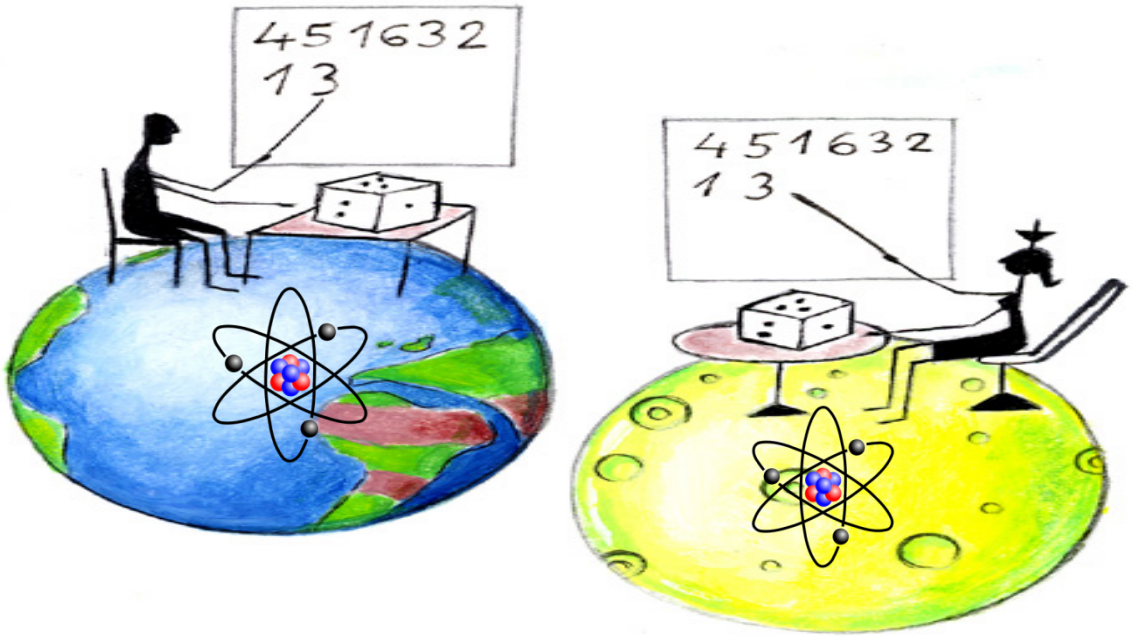
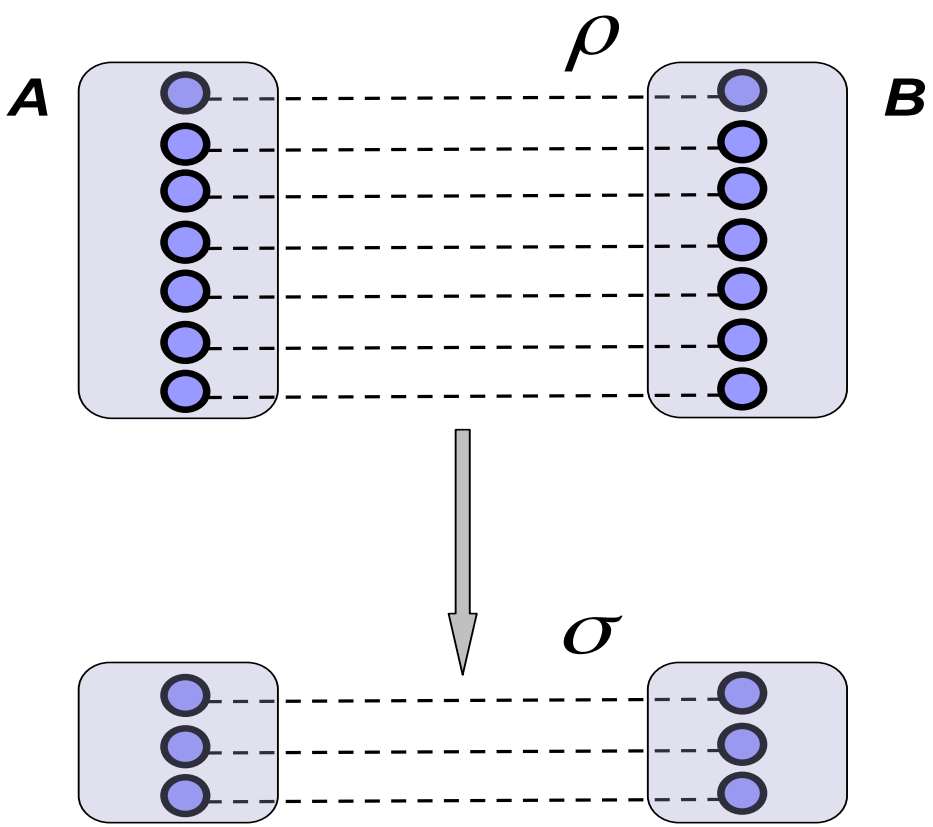


$$\rho \neq \sum_i p_i \rho_A^{\,i} \otimes \rho_B^i$$

Cannot be created by local operations and classical communication (LOCC)

LOCC asymptotic entanglement transformations

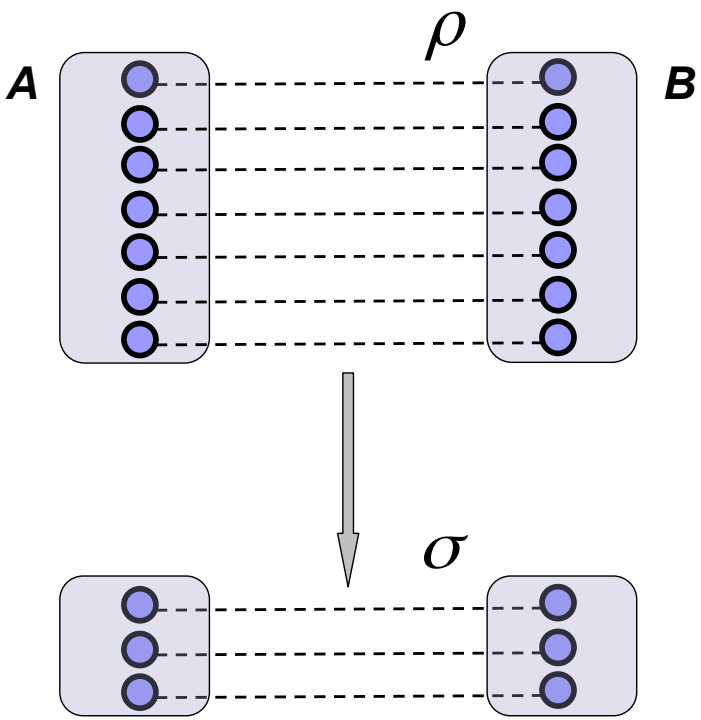


$$\rho^{\otimes n} \rightarrow_{LOCC} \sigma_n \approx \sigma^{\otimes k_n}$$

LOCC asymptotic entanglement transformations

$$\rho \rightarrow_{LOCC} \sigma^{\otimes \mu} \Leftrightarrow$$

$$\lim_{n\to\infty} \min_{\Lambda_n \in LOCC} D(\Lambda_n(\rho^{\otimes n}),\ \sigma^{\otimes \mu_n}) = 0,\ \lim_{n\to\infty} \frac{\mu_n}{n} = \mu$$



Bipartite pure state entanglement transformations

- Transformations are reversible (Bennett, Bernstein, Popescu, Schumacher 96)

$$|\psi\rangle^{\otimes E(\psi)} \rightarrow_{LOCC} |\varphi\rangle^{\otimes E(\varphi)}$$

$$|\varphi\rangle^{\otimes E(\varphi)} \rightarrow_{LOCC} |\psi\rangle^{\otimes E(\psi)}$$

- Unique entanglement measure

  Entropy of Entanglement: $E(\psi) = S(\psi_A)$

- The asymptotic limit is crucial!

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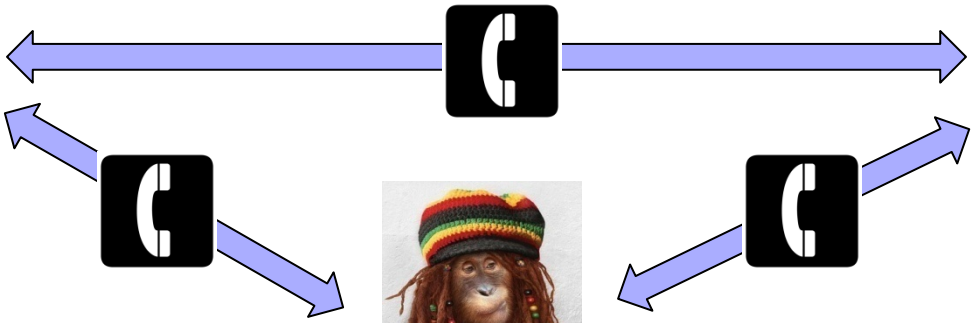
Multipartite pure state entanglement transformations

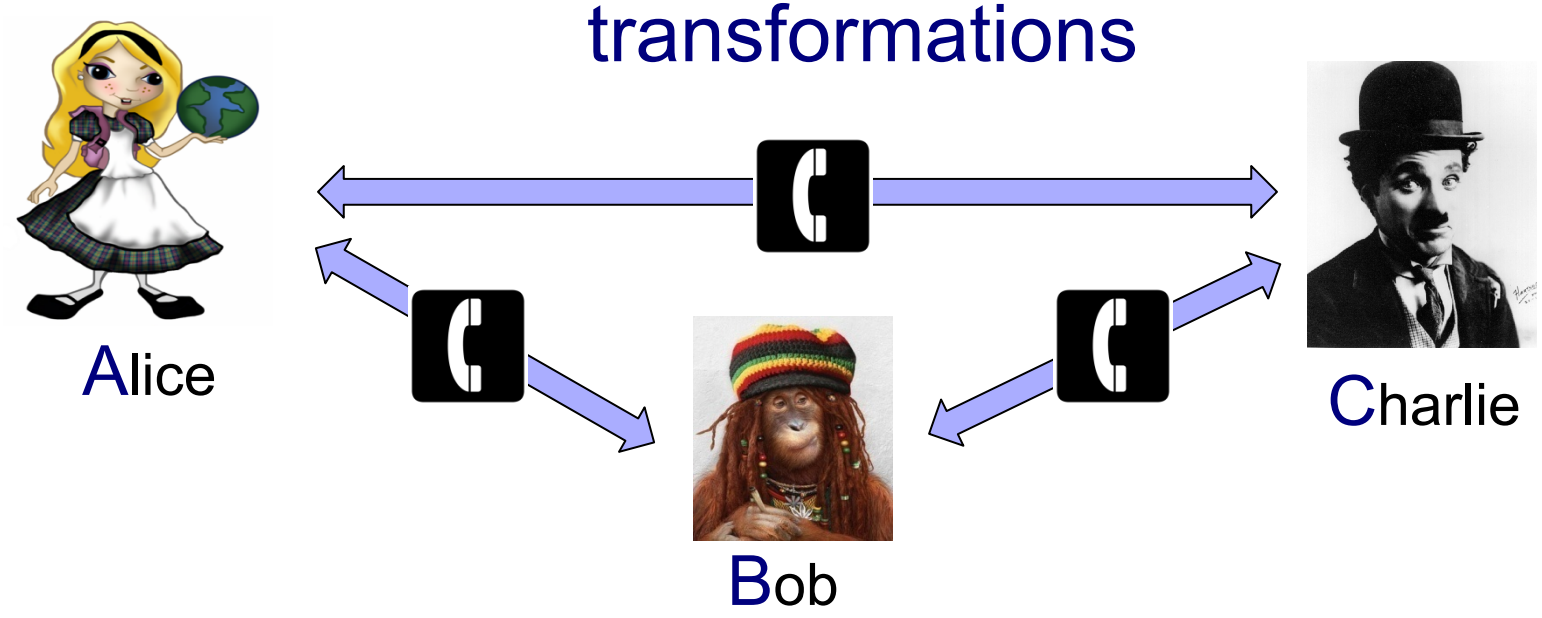
Alice



Bob

Charlie

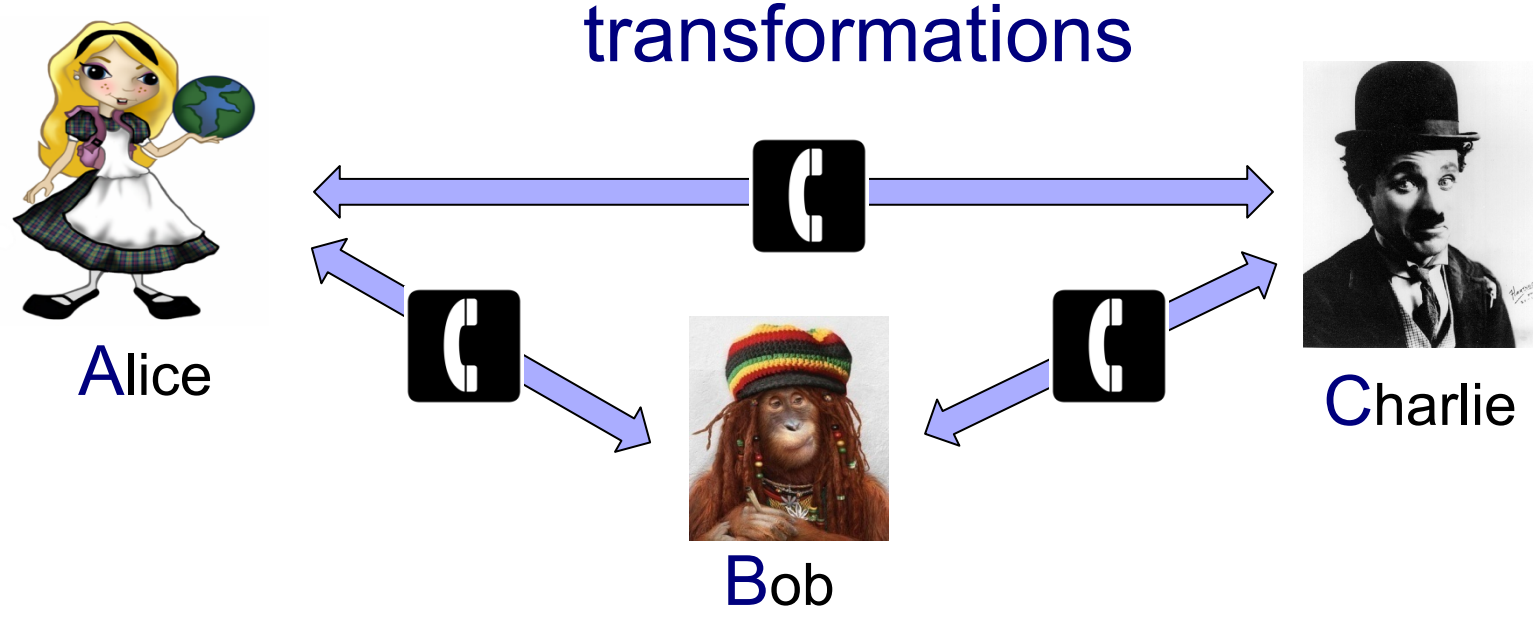
Multipartite pure state entanglement transformations



- There are inequivalent types of entanglement

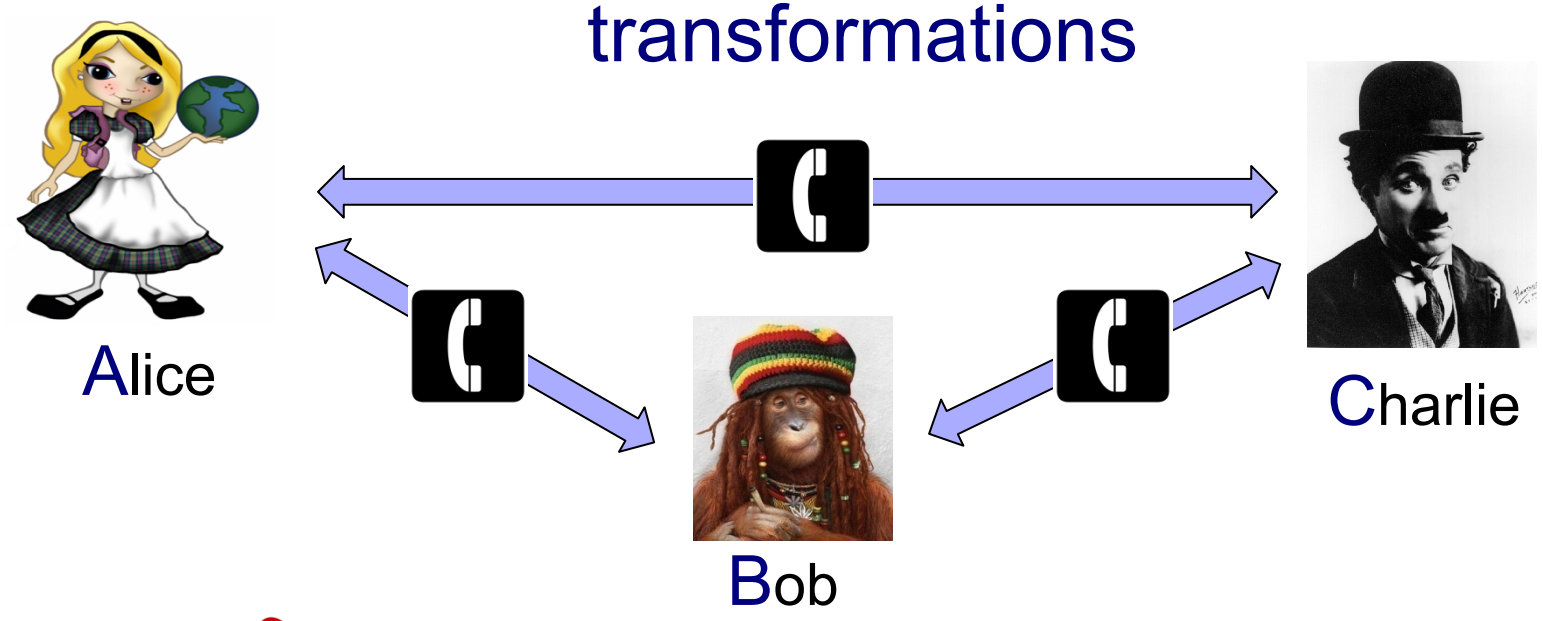
$$|EPR\rangle_{AB} \otimes |0\rangle_C \not\leftrightarrow |EPR\rangle_{AC} \otimes |0\rangle_B \not\leftrightarrow |EPR\rangle_{BC} \otimes |0\rangle_A$$

Multipartite pure state entanglement transformations



- $|\psi\rangle_{ABC} \leftrightarrow_{LOCC} |EPR\rangle_{AB}^{\otimes E_{AB}} \otimes |EPR\rangle_{AC}^{\otimes E_{AC}} \otimes |EPR\rangle_{BC}^{\otimes E_{BC}}$???

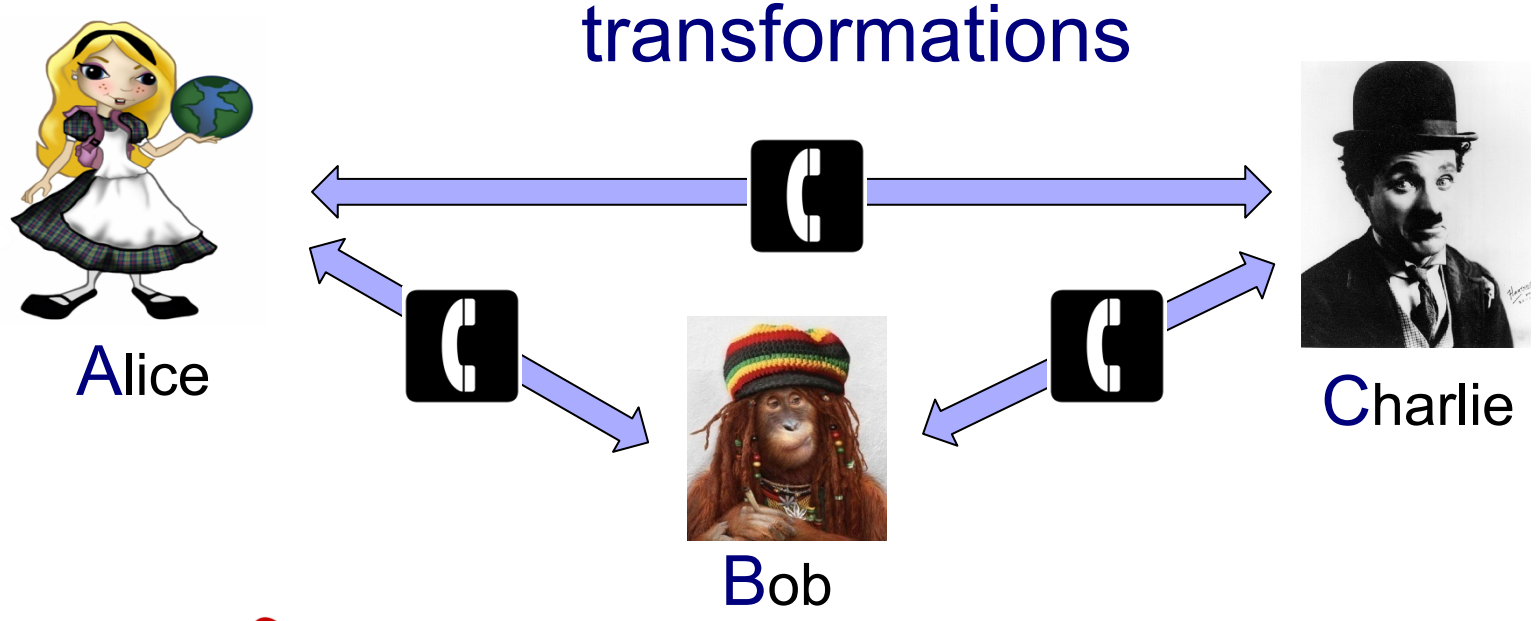
Multipartite pure state entanglement transformations



- $|GHZ\rangle_{ABC} \not\leftrightarrow_{LOCC} |EPR\rangle_{AB}^{\otimes E_{AB}} \otimes |EPR\rangle_{AC}^{\otimes E_{AC}} \otimes |EPR\rangle_{BC}^{\otimes E_{BC}}$

(Linden, Popescu, Schumacher, Westmoreland 99)

Multipartite pure state entanglement transformations



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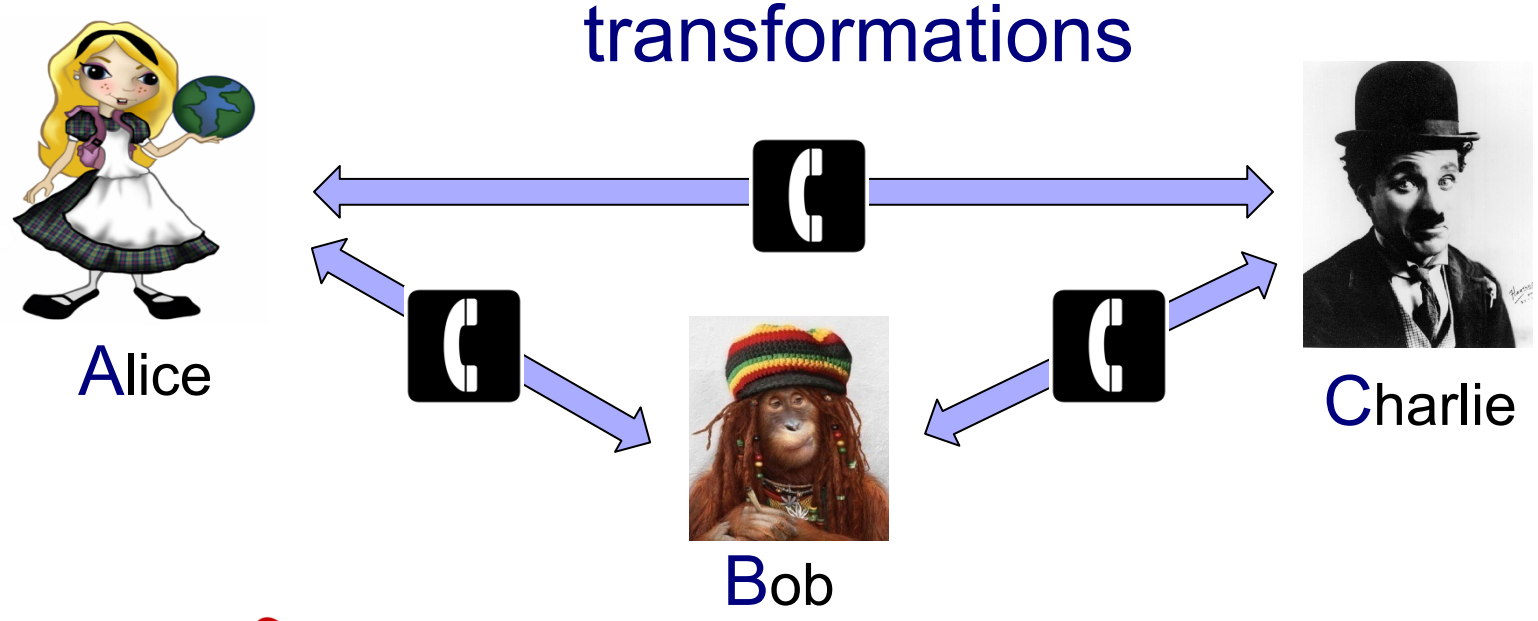
Multipartite pure state entanglement transformations



- $|GHZ\rangle_{ABC} \not\xrightarrow{LOCC} |EPR\rangle_{AB}^{\otimes r_{AB}} \otimes |EPR\rangle_{AC}^{\otimes r_{AB}} \otimes |EPR\rangle_{AC}^{\otimes r_{AB}}$

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(Acin, Vidal, Cirac 02)

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District gets set for MREG
5 Jan 2008 ... Pune, January 4
Employment Guarantee Scheme
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MREGS picks up in Hingoli
After a poor start, the Maharashtra Rural Employment Guarantee Scheme (MREGS) has picked up tempo in Hingoli district of Marathwada, due to efforts of a proactive

Read more>>

CSO

Village campaigns expose poor state of MREGS
A PACS Programme campaign shows that the National Rural Employment Guarantee Scheme (NREGS) is being poorly implemented in many parts of Maharashtra.

e>>

THIS TALK GOAL: To expose poor state of MREGS (Minimum Reversible Entanglement Generating Set)

- T

(B ett, Popescu, Rohrlich, Smolin, Thapliyal 99)

MREGS

- We say $\left\{ \left| \varphi_k \right\rangle_{ABC} \right\}_{k=1}^{N}$ is a MREGS if

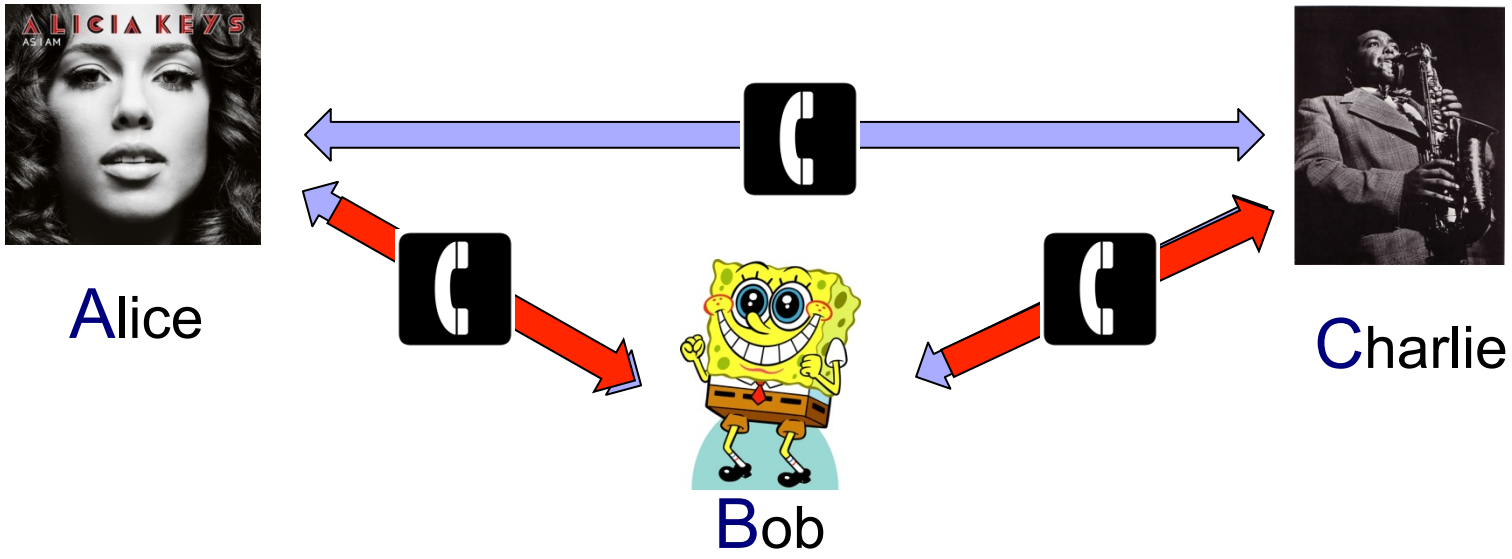
$$\left| \psi \right\rangle_{ABC} \leftrightarrow_{LOCC} \bigotimes_{k=1}^{N} \left| \varphi_k \right\rangle_{ABC}^{\otimes r_k}$$

and *N* is the minimum number for which this holds

- Conjecture: There is no finite MREGS

- Rest of the talk: two arguments supporting the conjecture

No MREGS under 1-LOCC



- Theo 1: Under 1-way LOCC (Alice to Bob and Bob to Charlie), there is no finite MREGS for tripartite states of qubits (2 x 2 x 2 states)

Sometimes1-LOCC is enough

- For *bipartite* pure states 1-LOCC is enough (Bennett, Bernstein, Popescu, Schumacher 96)

- For the only two *known* classes of multipartite states for which a MREGS exist, 1-LOCC is enough

  1. Schmidt decomposable states (Bennett, Popescu, Rohrlich, Smolin, Thapliyal): $\sum_k \alpha_k |a_k, b_k, c_k\rangle$

  2. The family (Vidal, Dür, Cirac): $c_0|000\rangle + c_1|1\rangle\left(|11\rangle + |22\rangle\right)$

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From tripartite pure to bipartite mixed states

- (proof theo 1) Suppose there *is* a finite MREGS:

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- Thus there must exist a *mixed* bipartite MREGS under 1-LOCC... but we can show it cannot exist!

k-Extendible States

- We say a state ψ_{AB} is *k*-extendible if there is a state $\psi_{AB_1 \ldots B_k}$ such that $\psi_{ABj} = \psi_{AB}$ for all *j*

- Only separable states are *k*-extendable for every *k* (Raggio and Werner 89)

- ...and for every *k* there are *k*-extendable *yet* entangled states

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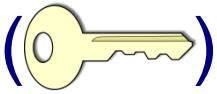
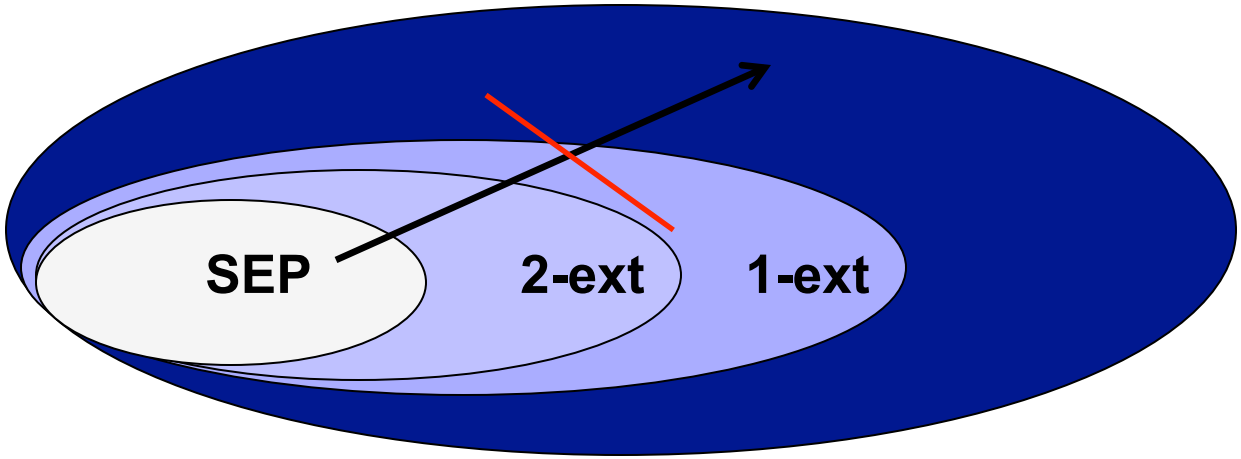
k-Extendability is preserved by 1-LOCC

- () the set of *k*-extendible states is preserved under 1-LOCC and under tensoring



- Let k_{max} be the max. k such that all $\varphi_{k,AB}$ are k-ext.
- Take ψ_{AB} to be a ($k_{max}+1$)-ext. state. **QED**

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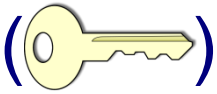
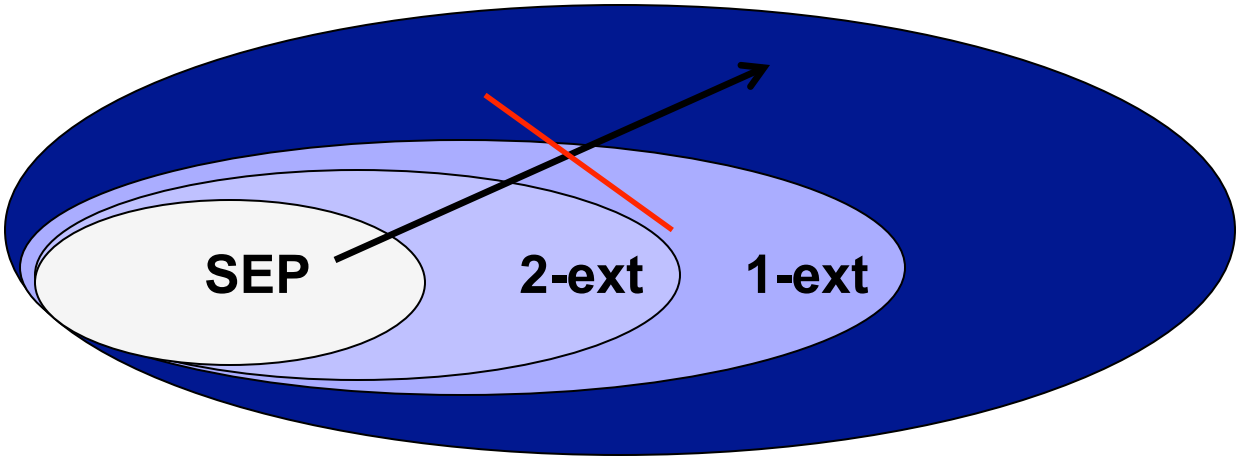
- () the set of *k*-extendible states is preserved under 1-LOCC and under tensoring



$$\psi_{AB} \leftrightarrow 1\text{-}LOCC \bigotimes_{k=1}^{N} \varphi_{k,AB}^{\otimes r_k}$$

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Intermezzo: Interconverting two useless channels

- We just saw there are infinitely many different types of 1-LOCC undistillable states (namely, *k*-extendible states)

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- We just saw there are infinitely many different types of 1-LOCC undistillable states (namely, *k*-extendible states)

- By Jamiolkowski/Choi isomorphism, we find there are infinitely many inequivalent classes of zero-capacity channels too: There is an infinite sequence $\left\{\Lambda_k : H_A \to H_B\right\}_{k\in N}$ such that for all *n*,

$$\Lambda_{k+1}^{\otimes n} \not\to \Lambda_k$$

- E.g. Erasure channel with 1/n erasure probability.

No MREGS under LOCC?

Alice

Bob

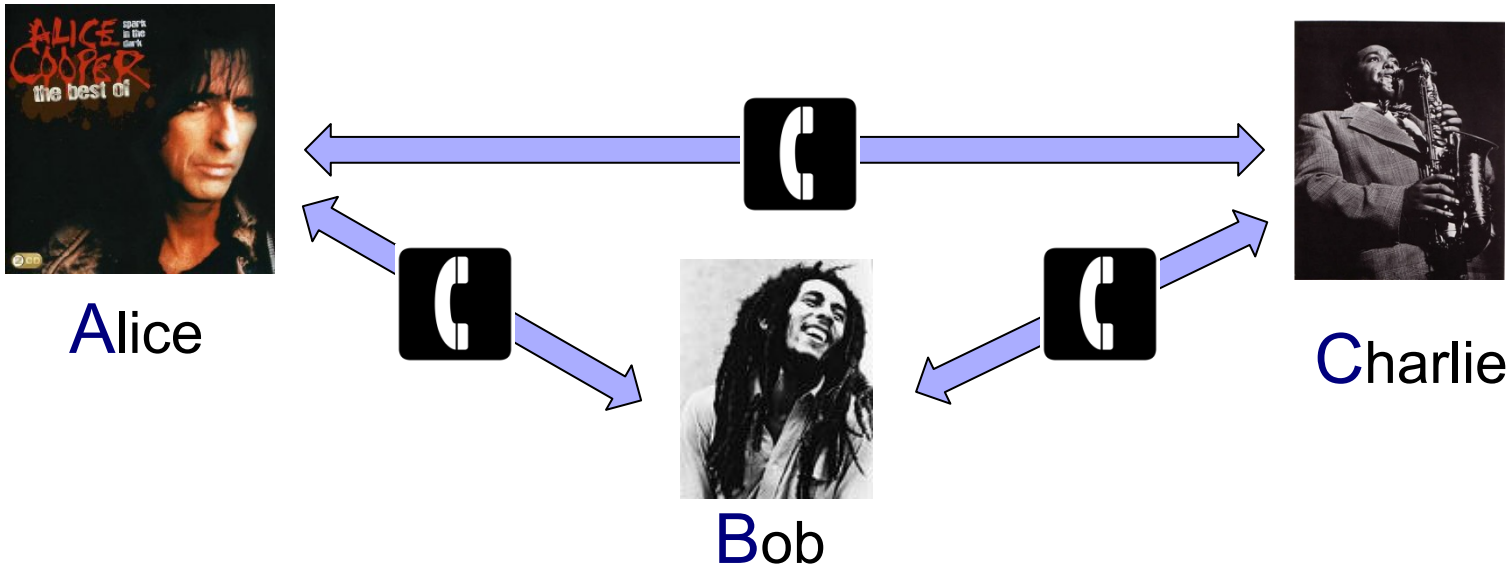
Charlie

In the general case of unrestricted LOCC communication among the parties, all proof fails completely

Reason: general LOCC do not preserve the sets of *k*-extendible states.

Is there a good replacement for *k*-ext. states??

No MREGS under LOCC?



- Theo 2: Assuming a certain *conjecture about bipartite mixed states*, there is no MREGS for tripartite systems (already for 3 x 3 x 2).

The Conjecture

There are infinitely many inequivalent types of bipartite entanglement, i.e. $\{\rho_k\}_{k=1}^{\infty}$ such that for all *k*, *n*

$$\rho_k^{\otimes n} \not\mapsto \rho_{k+1}$$

Bound Entanglement

- We know *two* classes:

1. distillable states: $\rho^{\otimes n} \rightarrow \left|EPR\right\rangle\left\langle EPR\right|$

2. bound entanglement: $\rho^{\otimes n} \not\rightarrow \left|EPR\right\rangle\left\langle EPR\right|$
   (Horodecki, Horodecki, Horodecki 98)

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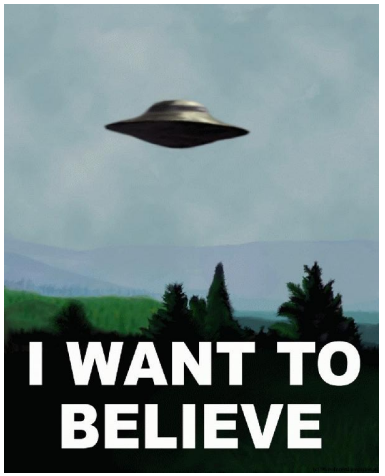
- The conjecture is really about bound entanglement: We want a sequence $\left\{\rho_k\right\}_{k=1}^{\infty}$ of bound entangled states s.t.

$$\rho_k^{\otimes n} \not\rightarrow \rho_{k+1}$$

Supporting the Conjecture

- Why is it reasonable?

"The same evidences that we have for bound entangled Werner states with a non-positive partial transpose apply to the conjecture."



- Namely

Let $\rho_p = p\omega^- + (1-p)\omega^+$

For any *n* there is a non-zero interval (a, b) for which

$$\rho_p^{\otimes n} \not\mapsto \rho_{p+\varepsilon} \quad \forall\ p \in (a,\ b),\ \varepsilon > 0$$

But the interval *might* shrink to zero when *n* grows....

From Tripartite Pure to Bipartite Mixed, Again

- (proof sketch) Assume there is a finite MREGS

$$|\psi\rangle_{ABC} \leftrightarrow_{LOCC} \bigotimes_{k=1}^{N} |\varphi_k\rangle_{ABC}^{\otimes r_k}$$

- As before, the strategy is to relate it to a problem about bipartite mixed states...

- The basic idea dates back to Linden, Popescu, Schumacher, Westmoreland 99 who used it to show that the GHZ state is not equivalent to EPR pairs.

The Relative Entropy Must Be Preserved

- Linden, Popescu, Schumacher, Westmoreland proved that

$$|\psi\rangle_{ABC} \leftrightarrow_{LOCC} \bigotimes_{k=1}^{N} |\varphi_k\rangle_{ABC}^{\otimes r_k}$$

implies

$$E_R^{\infty}(\psi_{AB}) = E_R^{\infty}\left(\bigotimes_k \varphi_{k,AB}^{\otimes r_k}\right)$$

where the regularized relative entropy of entanglement reads

$$E_R^{\infty}(\psi_{AB}) := \lim_{n\to\infty} \frac{1}{n} \min_{\sigma\in S} S(\psi_{AB}^{\otimes n} \,\|\, \sigma)$$

Some New Entanglement Measures

- We do the same, but for infinitely many related measures. Let $\{\rho_k\}_{k=1}^{\infty}$ s.t. $\rho_k^{\otimes n} \not\mapsto \rho_{k+1}$ and define

$$E_k^{\infty}(\psi_{AB}) := \lim_{n\to\infty} \frac{1}{n} \min_{\sigma \in O(\rho_k)} S(\psi_{AB}^{\otimes n} \| \sigma)$$

Where the asymptotic orbit of ρ_k reads

$$O(\rho_k) := \left\{ \pi : \pi = \Omega(\rho^{\otimes n}) / tr(\Omega(\rho^{\otimes n})),\ \Omega \in SLOCC \right\}$$

The Relative Entropies Must Be Preserved

- We can show: If

$$|\psi\rangle_{ABC} \leftrightarrow_{LOCC} \bigotimes_{k=1}^{N} |\varphi_k\rangle_{ABC}^{\otimes r_k}$$

For all k

$$E_k^{\infty}(\psi_{AB}) = E_k^{\infty}\left(\bigotimes_j \varphi_{j,AB}^{\otimes r_j}\right)$$

The proof is an easy adaptation of the result of Popescu et al. (the measures have all the nice properties: monotone under LOCC, non-lockable, asymptotically continuous, subadditive, ...)

One Last Nice Property

- From a result of B., Plenio 09 on extensions of quantum Stein's Lemma we find

$$\sigma \notin O(\rho_k) \Rightarrow E_{k+1}^{\infty}(\sigma) > 0$$

- The proof is the same as for the regularized relative entropy of entanglement. See also M. Piani 09.

The final part

- Assuming MREGS, we have

1) $$0 = E_k^{\infty}(\rho_k) = E_k^{\infty}\left(\bigotimes_j \varphi_{j,AB}^{\otimes r_j}\right) \geq r_j E_k^{\infty}(\varphi_{j,AB})$$

$$\left|\psi_{\rho_k}\right\rangle_{ABC} \leftrightarrow_{LOCC} \bigotimes_{j=1}^{N} \left|\varphi_j\right\rangle_{ABC}^{\otimes r_j}$$

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- Therefore for every *k* there is a *j* such that

$$E_k^{\infty}(\varphi_{j,AB}) = 0 \ , E_{k+1}^{\infty}(\varphi_{j,AB}) > 0$$

- But this cannot be, as by assumption there are only finitely many *j*. **QED**

Conclusion and Open Questions

- We showed that there is no finite MREGS
  1. Under 1-LOCC
  2. Assuming there are infinitely many classes of bipartite bound entanglement

- Open questions:
  1. Is backward classical communication helpful?
  2. Can we prove there are more than 2 types of of bound entanglement? Related to **NPPT**BE!
  3. Is there a finite MREGS?

Thank you!