Physics 125a

Problem Set 5, Due Wed. Nov 23, 2016

Problem 1

(a) Two identical bosons are found to be in the states $|\phi\rangle$ and $\psi\rangle$. Write down the normalized state vector describing the system when $\langle\phi|\psi\rangle \neq 0$.

(b) A particle moves in a potential $V(x) = V_0 \sin(2\pi x/a)$. The potential is invariant under the translations: $x \to x + ka$ where k is an integer. Is momentum conserved and why?

Problem 2

Suppose we prepare the combined state of two spin 1/2 particles A and B (which are not identical)

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}} \left(|\hat{\mathbf{z}};\uparrow\rangle_A |\hat{\mathbf{z}};\uparrow\rangle_B + |\hat{\mathbf{z}};\downarrow\rangle_A |\hat{\mathbf{z}};\downarrow\rangle_B \right)$$

where $|\hat{\mathbf{z}} :\uparrow\rangle_A$ denotes that particle A has spin up along the $\mathbf{z}$ axis, etc. Note we are suppressing all the quantum numbers except the spin quantum numbers here.

(a) Show that

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}} \left(|\hat{\mathbf{x}};\uparrow\rangle_A |\hat{\mathbf{x}};\uparrow\rangle_B + |\hat{\mathbf{x}};\downarrow\rangle_A |\hat{\mathbf{x}};\downarrow\rangle_B \right)$$

where $|\hat{\mathbf{x}};\uparrow\rangle_A$ denotes that particle A has spin up along the $\mathbf{x}$ axis, etc.

Alice and Bob are friends. Bob has the bright idea to use this state to do faster than light communication: he prepares an ensemble with many copies of the state $|\Psi_{AB}\rangle$. and then carefully separates the particles A and B in each pair, keeping particles B in Pasadena and giving particles A to Alice who travels to Pluto. When he wants to send the message Yes he measures his particles spins along the $\mathbf{z}$ direction, which immediately prepares Alice's particles in an ensemble of the states $|\hat{\mathbf{z}};\uparrow\rangle$ and $|\hat{\mathbf{z}};\downarrow\rangle$. For the message No he measures his particles spins along the $\mathbf{x}$ direction, which immediately prepares Alice's particles in an ensemble of the states $|\hat{\mathbf{x}}:\uparrow\rangle$ and $|\hat{\mathbf{x}};\downarrow\rangle$. Alice then measures her spins to see which state Bob been prepared and hence deduce his message.

(b) Let us suppose that Alice measures her spins along the $\mathbf{z}$ axis. If Bob has sent the message Yes or No show that Alice will measure spin up half the time and spin down half the time. Similarly if Alice measures her spins along the $\mathbf{x}$ axis. Silly Bob, well at least he isn't stuck on Pluto.

Problem 3

Show that $\langle J_x\rangle = \langle J_y\rangle = 0$ in the states $|j,m\rangle$ and that in these states

$$\langle J_x^2\rangle = \langle J_y^2\rangle = \frac{1}{2}\hbar^2\left[j(j+1) - m^2\right].$$

Problem 4

Consider a spinless particle in a state represented by the wave-function

$$\psi(x,y,z) = C(x+y+2z)\exp(-\gamma r)$$

where $r = \sqrt{x^2+y^2+z^2}$.

(a) What is the total orbital angular momentum of the particle (i.e., the quantum number ℓ)?

(b) What is the expectation value of the z component of the orbital angular momentum?

(c) If the z-component of the orbital angular momentum L_z is measured what is the probability that the result would be $\hbar$.

Hint: You might find the explicit expressions for the first few spherical harmonics useful.