Physics 125b

Problem Set 5, Due Friday Mar. 10, 2017

Problem 1

Consider two spin 1/2 particles a and b. Particle a is the beam particle and particle b is the target particle. Particle b is very heavy so we treat is as at rest at the origin $r = 0$.

(a) Derive an expression for the differential cross section in the Born approximation when spin effects are included. Take the initial state to be $|\vec{p}_i\rangle \otimes |\chi_i\rangle$ and the final state to be $|\vec{p}_f\rangle \otimes |\chi_f\rangle$. Here $\vec{p}_{i,f}$ are the initial and final momenta of particle a and $\chi_{i,f}$ is the spin state of particles a and b before and after the scattering. Assume the potential is a product of a terms that depend on spin and position.

(b) Suppose the beam and target particles initially have opposite spins along the beam direction which we take to be along the z-axis. More explicitly the initial spins are eigenstates of S_a^z and S_b^z with eigenvalues $\hbar/2$ and $-\hbar/2$. Calculate in the Born approximation the differential cross section using the potential

$$V(r) = \vec{S}_a \cdot \vec{S}_b f(r),$$

where r is the radial coordinate and $\vec{S}_{a,b}$ are the spin operators for particles a and b. You can express your answer in terms of the Fourier transform of $f(r)$. Remember to sum over the possible final states of the beam and target particles since the spins of the beam and target particle are not being measured after the scattering.

Problem 2

This is a continuation of problem 1(**b**). Now suppose we also measure the final (*i.e.* after the scattering) spin of particle a. What is the differential cross section for it to be in an eigenstate of S_a^z with eigenstate $\hbar/2$.

Problem 3

In class we found that the Green's function satisfying

$$(\nabla^2 + k^2)G^0(\vec{r}) = \delta^2(\vec{r})$$

is useful in scattering. When we tried to get an explicit expression for $G^0(\vec{r})$ a divergent integral was encountered which we regulated by adding $i\epsilon$, $(\epsilon > 0)$ to the denominator. Suppose we regulated it by adding $-i\epsilon$. Evaluate the Green's function

$$G^0(\vec{r}) = \lim_{\epsilon \to 0} \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{k^2 - q^2 - i\epsilon}$$

Comment on the difference between this Green's function and the one we found in class.

Problem 4

Consider identical non-interacting non relativisitic massless bosons. In field theory their Hamiltonian is

$$H = \frac{\hbar^2}{2m} \int d^3x \vec{\nabla}\phi^\dagger(\vec{x}) \cdot \vec{\nabla}\phi(\vec{x})$$

Consider the two particle state $|\vec{k}_1, \vec{k}_2\rangle = b^\dagger(\vec{k}_1)b^\dagger(\vec{k}_2)|0\rangle$. This state consists of two particles with momentum $\hbar\vec{k}_1$ and $\hbar\vec{k}_2$.

(a) Show that this state is symmetric under exchange of the quantum numbers of the two particles, *i.e.*, $\vec{k}_1$ and $\vec{k}_2$.

(b) Show that it is an eigenstate of the Hamiltonian and find its eigenvalue.

(c) What is value of the inner product, $\langle \vec{k}_1', \vec{k}_2' | \vec{k}_1, \vec{k}_2 \rangle$.