

Exponential Decay of Correlations Implies Area Law: Single-Shot Techniques

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Based on joint work with Michał Horodecki

Arxiv:1206.2947

Beyond i.i.d. in Info. Theory, Cambridge 08/01/2012

This talk:

- Go *beyond* the *i.i.d.* simplification used in the previous talk.

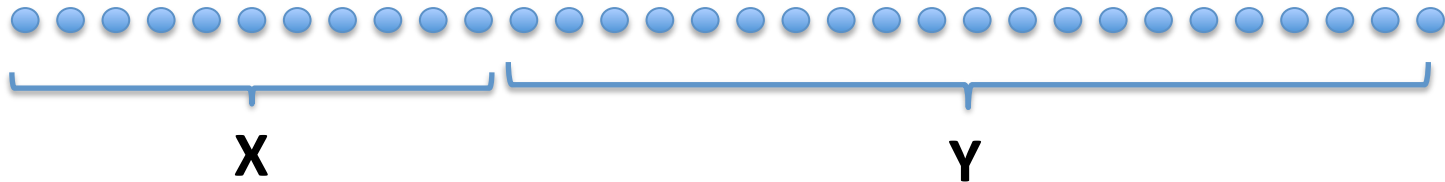
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- Contrast two views of single-shot info theory:
 1. It's the right info theory, iid-ness is an abstraction

This talk:

- **Go *beyond* the *i.i.d.* simplification used in the previous talk.**
- **Contrast two views of single-shot info theory:**
 - 1. It's the right info theory, iid-ness is an abstraction**
 - 2. It only gives a trivial reformulation of the problems**
(e.g. H_{\max} gives the compression rate or is it *defined* as the compression rate?)

The result



Thm 1 (B., Horodecki '12) If $|\psi\rangle_{1,\dots,n}$ has ξ -EDC, then for every $X = [1, r]$,

$$H_{\max}^{2^{-\Omega(l)}}(X) \leq 2^{O(\xi \log(\xi))} + l$$

$$\text{Cor}(A : C) := \max_{\|X\|, \|Y\| \leq 1} \left| \text{tr} \left((X \otimes Y) (\rho_{AC} - \rho_A \otimes \rho_C) \right) \right|$$

ξ -EDC (**E**xponential **D**ecay of **C**orrelations): For all regions A and C separated by l sites,

$$\text{Cor}(A : C) \leq 2^{-l/\xi}$$

The proof sketch...

...was based on two ingredients:

1. State Merging Protocol (Horodecki, Oppenheim, Winter '05)
2. “Saturation of Mutual Information” (based on Hastings '07)

But we only have **one** copy of the state...

Single-shot info theory to rescue?

Single-Shot Entropies

1. Conditional min-entropy (Renner '05)

$$H_{\min}(A|B)_{\rho} := \max_{\sigma} \sup \left\{ \lambda : 2^{-\lambda} I_A \otimes \sigma_B \geq \rho_{AB} \right\}$$

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2. Smooth conditional min entropy

$$H_{\min}^{\varepsilon}(A|B)_{\rho} := \max_{\rho' \approx_{\varepsilon} \rho} H_{\min}(A|B)_{\rho'}$$

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$$H_{\max}^{\varepsilon}(A|C) := -H_{\min}^{\varepsilon}(A|B), \text{ for } |\psi\rangle_{ABC}$$

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$$H_{\max}^{\varepsilon}(A|C) := -H_{\min}^{\varepsilon}(A|B), \text{ for } |\psi\rangle_{ABC}$$

4. Smooth max mutual info

$$I_{\max}^{\varepsilon}(A:B) := H_{\max}^{\varepsilon}(A) - H_{\min}^{\varepsilon}(A|B)$$

Single-Shot Counterparts 1: Correlations vs Entropies

From i.i.d. merging: For $|\psi\rangle_{ABC}$

$$H(C) > H(B) \Rightarrow \text{Cor}(A : C) \geq \exp(-I(A : B))$$

Lemma: For $|\psi\rangle_{ABC}$ and $\delta, \nu > 0$,

1. $-H_{\max}^{\delta}(A|C) \geq -2 \log \delta + 5 \Rightarrow$

$$\text{Cor}(A : C) \geq \exp(-I_{\max}^{\delta}(A : B) + 2 \log \delta)$$

2. $H_{\max}^{\nu}(C) \geq 3H_{\max}^{\delta}(B) \Rightarrow$

$$\text{Cor}(A : C) \geq \exp(-3H_{\max}^{\delta}(B) - 6 \log(\delta^{-2} + \nu^{-2}))$$

Single-Shot Counterparts 1:

Correlation

$$H(C) - H(AC) = -H(A|C) > 0$$

From i.i.d. merging: For $|\psi\rangle_{ABC}$

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Single-Shot Counterparts 1:

Correlations

From i.i.d. merging: For

$$H(C) > H(B) \Rightarrow$$

Independent bounds since

$$I_{\max}^{\delta}(A : B) \leq c H_{\max}^{\varepsilon}(B)$$

Lemma: For $|\psi\rangle_{ABC}$ and $\delta, \nu > 0$,

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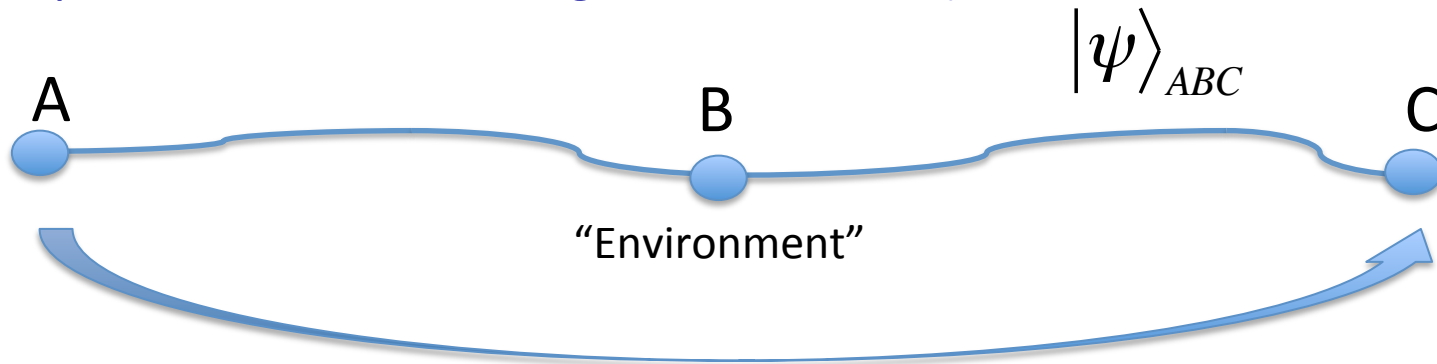
Single-Shot Merging

$$-H_{\max}^{\delta}(A|C) \geq -2 \log \delta + 5 \Rightarrow$$

$$\text{Cor}(A : C) \geq \exp(-I_{\max}^{\delta}(A : B) + 2 \log \delta)$$

Follows from single-shot state merging:

(Dupuis, Berta, Wullschleger, Renner '10)



EPR rate: $-H_{\max}^{\delta}(A|C) + 2 \log \delta - 5 \Leftrightarrow -H(A|C)$

C. Com. Cost: $I_{\max}^{\delta}(A : B) - 2 \log \delta \Leftrightarrow I(A : B)$

Beyond Single-Shot Merging

$$H_{\max}^{\nu}(C) \geq 3H_{\max}^{\delta}(B) \Rightarrow$$

$$\text{Cor}(A : C) \geq \exp(-3H_{\max}^{\delta}(B) - 6 \log(\delta^{-2} + \nu^{-2}))$$

Does **not** follow from single shot-merging. Might have

$$H_{\max}^{\nu}(C) \geq 3H_{\max}^{\delta}(B) \quad \text{but} \quad -H_{\max}^{\delta}(A|C) \ll 0 !!$$

We operate in a regime where one **cannot get EPR pairs**.
Still one **can get correlations by random measurements**

It **only happens in the single-shot regime**, asymptotically
either one gets EPR pairs between A and C or decouples A
from C

Noisy Correlations by Random Measurements

iid

$$H(A|C) = 0$$

Perfect correlations
EPR distillation

no correlations
(decoupling)

Single-shot

$$H_{\max}^{\delta}(A|C) = 0 \quad H_{\max}^{\delta}(A|B) = 0$$

Perfect correlations
EPR distillation

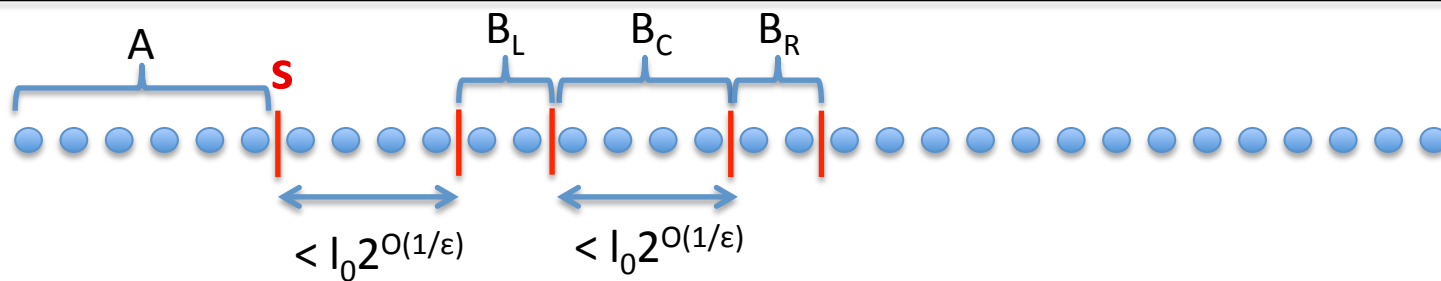
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Single-Shot Counterparts 2: Saturation Mutual Info

Given a site s , for all $l_0, \varepsilon > 0$ there is a region $B_{2l} := B_{L,l/2} B_{C,l} B_{R,l/2}$ of size $2l$ with $1 < l/l_0 < 2^{O(1/\varepsilon)}$ at a distance $< l_0 2^{O(1/\varepsilon)}$ from s s.t.

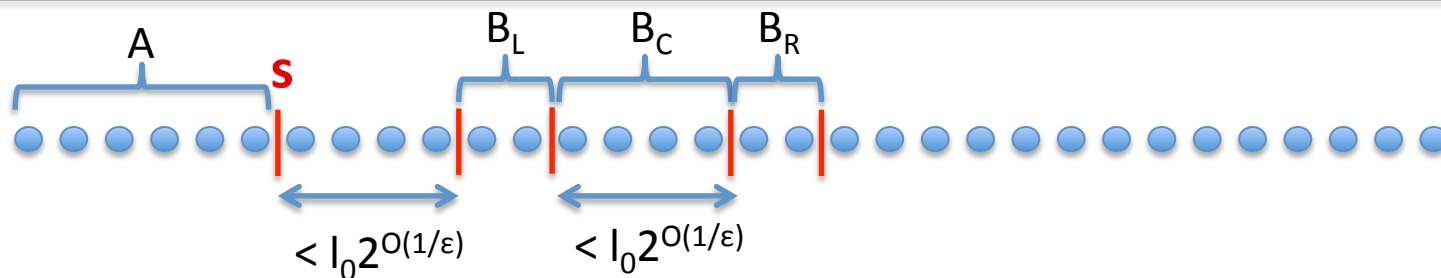
$$I(B_{C,l} : B_{L,l/2} B_{R,l/2}) < \varepsilon l$$



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Single-shot Let Ψ have ξ -EDC. Given a site s , for all $l_0, \varepsilon, \delta > 0$ there is a region $B_{2l} := B_{L,l/2} B_{C,l} B_{R,l/2}$ of size $2l$ with $1 < l/l_0 < \exp(\log(1/\varepsilon)/\varepsilon)$ at a distance $< l_0 \exp(\log(1/\varepsilon)/\varepsilon)$ from s s.t.

$$I_{\max}^{\delta}(B_{C,l} : B_{L,l/2} B_{R,l/2}) < 8\varepsilon l / \delta + \delta$$

Saturation von Neumann Mutual Information

Given a site s , for all $l_0, \varepsilon > 0$ there is a region $B_{2l} := B_{L,l/2} B_{C,l} B_{R,l/2}$ of size $2l$ with $1 < l/l_0 < 2^{O(1/\varepsilon)}$ at a distance $< l_0 2^{O(1/\varepsilon)}$ from s s.t.

$$I(B_{C,l} : B_{L,l/2} B_{R,l/2}) < \varepsilon l$$

$$I(B_C : B_L B_R) \leq \varepsilon l \Leftrightarrow H(B_C) + H(B_L B_R) - H(B_C B_L B_R) \leq \varepsilon l$$

Proof by contradiction: If it doesn't hold, for all l :

$$H(\underbrace{B_{C,l} B_{L,l} B_{R,l}}_{H(2l)}) \leq H(\underbrace{B_{C,l}}_{H(l)}) + H(\underbrace{B_{L,l} B_{R,l}}_{H(l)}) - \varepsilon l$$

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Interating:

$$H(2l) < 2H(l) - \varepsilon l < 4 H(l/2) - 2\varepsilon l < 8 H(l/4) - 3\varepsilon l < \dots$$

... which implies $H(l) < l H(2) - \varepsilon l / \log(l)$

Saturation max Mutual Information

Single-shot Let Ψ have ξ -EDC. Given a site s , for all $l_0, \varepsilon, \delta > 0$ there is a region $B_{2l} := B_{L,l/2} B_{C,l} B_{R,l/2}$ of size $2l$ with $1 < l/l_0 < \exp(\log(1/\varepsilon)/\varepsilon)$ at a distance $< l_0 \exp(\log(1/\varepsilon)/\varepsilon)$ from s s.t.

$$I_{\max}^{\delta}(B_{C,l} : B_{L,l/2} B_{R,l/2}) < 8\varepsilon l / \delta + \delta$$

We do **not** know how to prove it without exponential decay of correlations.

Proof **breaks down** completely:

1. I_{\max} not linear combination of entropies
2. Maybe use bound: $I_{\max}(A:B) < H_{\max}(A) + H_{\max}(B) - H_{\min}(AB)$
But **arbitrary gaps** between H_{\max} and H_{\min}

Saturation max Mutual Information

The idea: Use **EDC** to relate single-shot and i.i.d.

1. From the q. substate theorem (Jain, Radhakrishnan, Sen '07)

$$I_{\max}^{\delta}(B_C : B_L B_R) \leq \frac{1}{\delta} \left(H_{\max}^{\delta/2}(B_C) + H(B_L B_R) - H(B_C B_L B_R) \right) + \delta$$

Suffices to prove: $H_{\max}^{\delta/2}(B_{C,l}) + H(B_{L,l} B_{R,l}) \leq H(B_{C,l} B_{L,l} B_{R,l}) + \epsilon l$

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Suffices to prove:

$$\underbrace{H_{\max}^{\delta/2}(B_{C,l})}_{H_{\max}^{\delta/2}(l)} + \underbrace{H(B_{L,l} B_{R,l})}_{H(l)} \leq \underbrace{H(B_{C,l} B_{L,l} B_{R,l})}_{H(2l)} + \epsilon l$$

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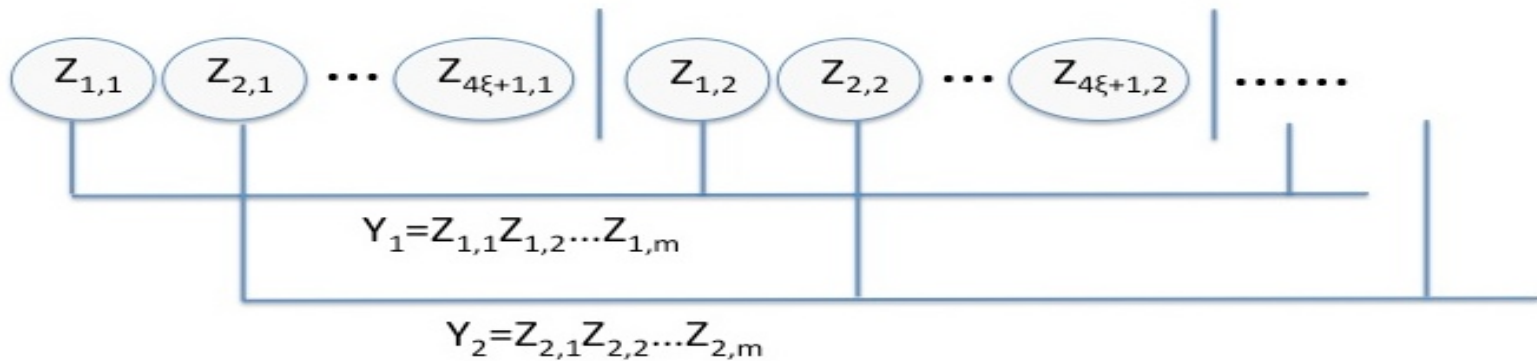
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2. **Suppose not:** For all l , $H(2l) \leq H_{\max}^{\delta/2}(l) + H(l) - \epsilon l$

We must relate $H_{\max}^{\delta/2}(l)$ to $H(l)$. **In general not possible**

But we have EDC...

Relating max to vNeumann entropy



By **subadditivity**: $H_{\max}^{\varepsilon}(B_{C,l}) \leq \sum H_{\max}^{\delta}(Y_k) + f(\varepsilon)$

But by **EDC**, $\left\| \rho_{Z_{k,1}, \dots, Z_{k,m}} - \rho_{Z_{k,1}} \otimes \dots \otimes \rho_{Z_{k,m}} \right\|_1$ is small.

Thus $H_{\max}^{\delta}(Y_k) \approx H_{\max}^{\delta}(\rho_{Z_{k,1}} \otimes \dots \otimes \rho_{Z_{k,m}})$

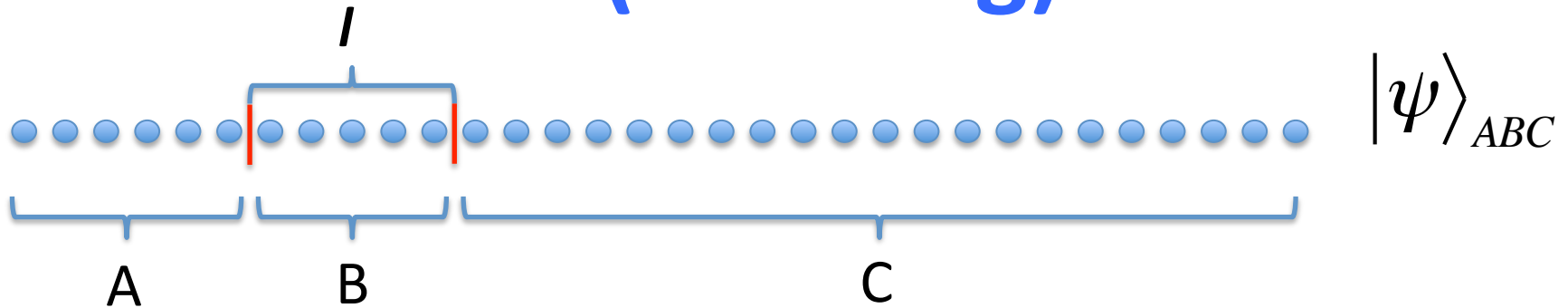
By **q. equipartition property** (Tomamichel, Colbeck, Renner '08)

$$H_{\max}^{\delta}(\rho_{Z_{k,1}} \otimes \dots \otimes \rho_{Z_{k,m}}) \approx H(\rho_{Z_{k,1}} \otimes \dots \otimes \rho_{Z_{k,m}})$$

Using Single-Shot Lemmas to show the main result

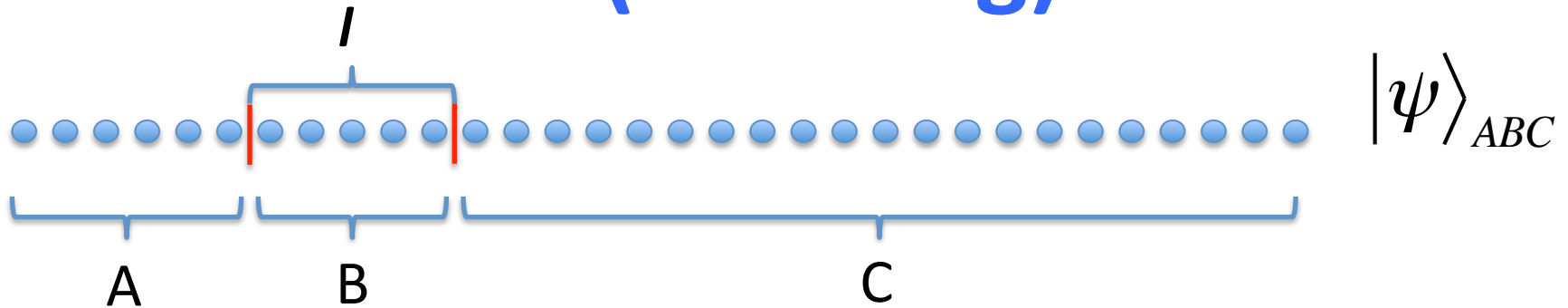
1. Area law from subvolume law
2. Getting subvolume law

Area Law from Subvolume Law (cheating)



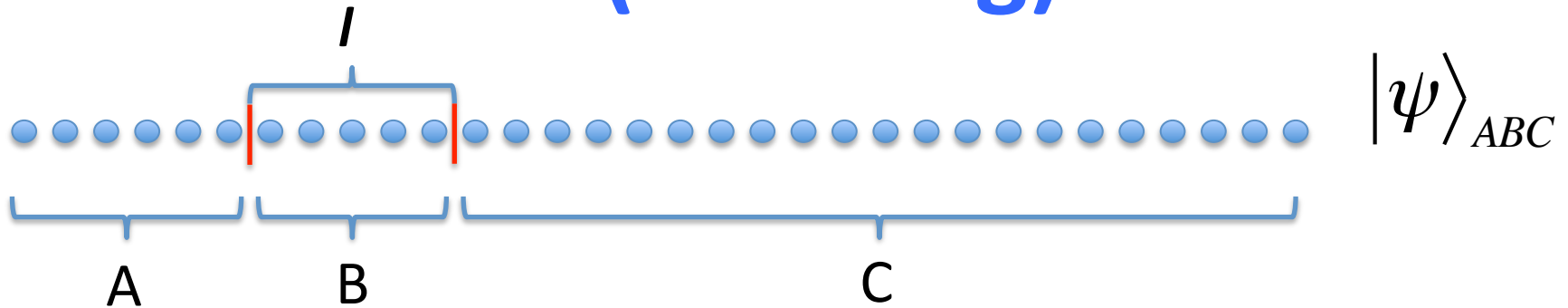
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$$H(C) \leq H(B) \iff Cor(A : C) < \exp(-I(A : B))$$

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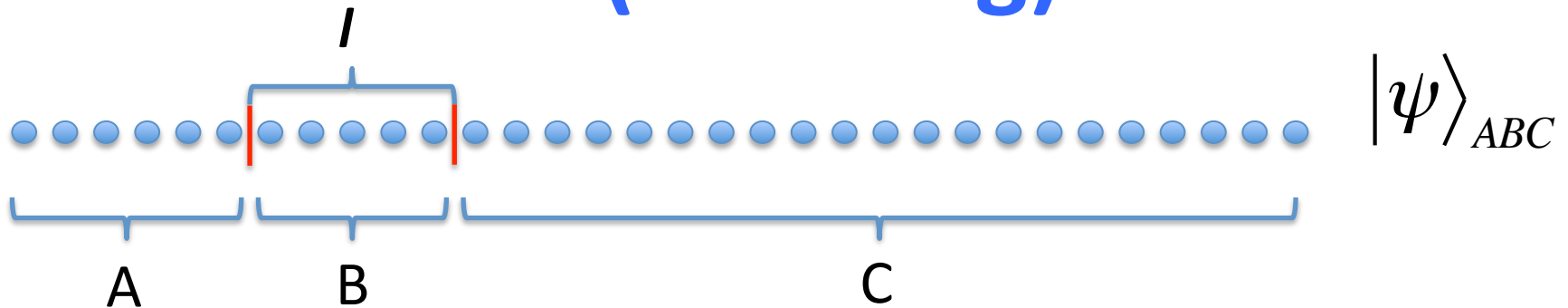


$$H(C) \leq H(B) \iff \text{Cor}(A:C) < \exp(-I(A:B))$$

Suppose $H(B) < 1/(4\xi)$. Since $I(A:B) < 2H(B) < 1/(2\xi)$, if state $|\psi\rangle_{ABC}$ has ξ -EDC then $\text{Cor}(A:C) < 2^{-1/\xi} < 2^{-I(A:B)}$

Thus: $H(C) < H(B)$: Area Law for C!

Area Law from Subvolume Law (cheating)



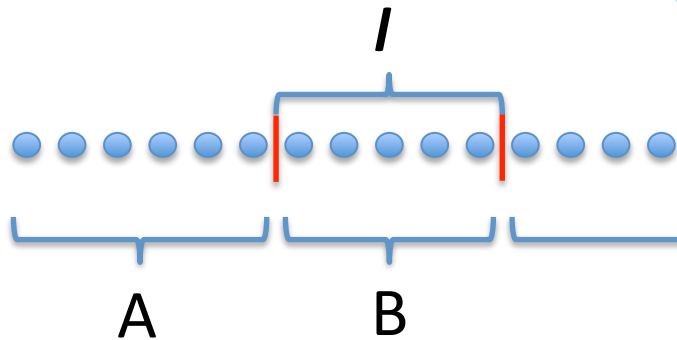
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It suffices to prove that nearby the boundary of C there is a region of size $< I_0 2^{O(\xi)}$ with entropy $< I/(4\xi)$

Area Law from Subvolume Law



Could consider single-shot merging. But remember

$$I_{\max}^{\delta}(A : B) \not\leq 2H_{\max}^{\epsilon}(B)$$

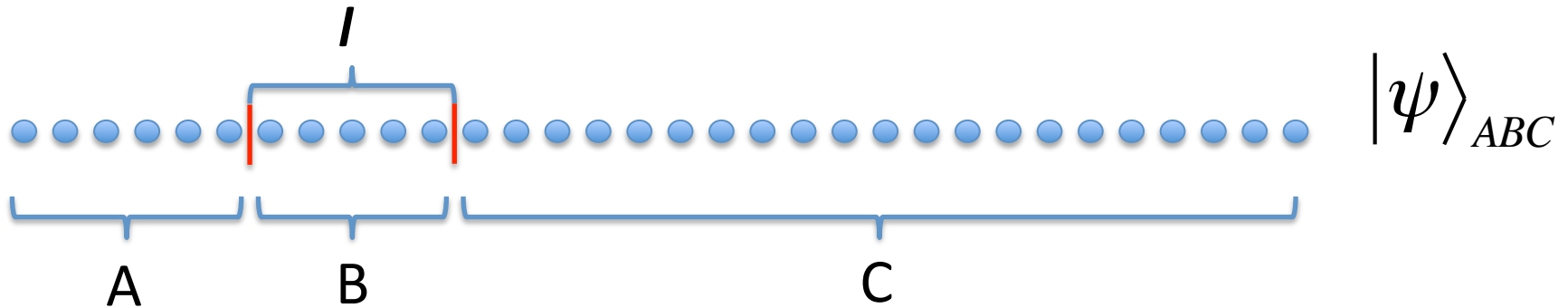
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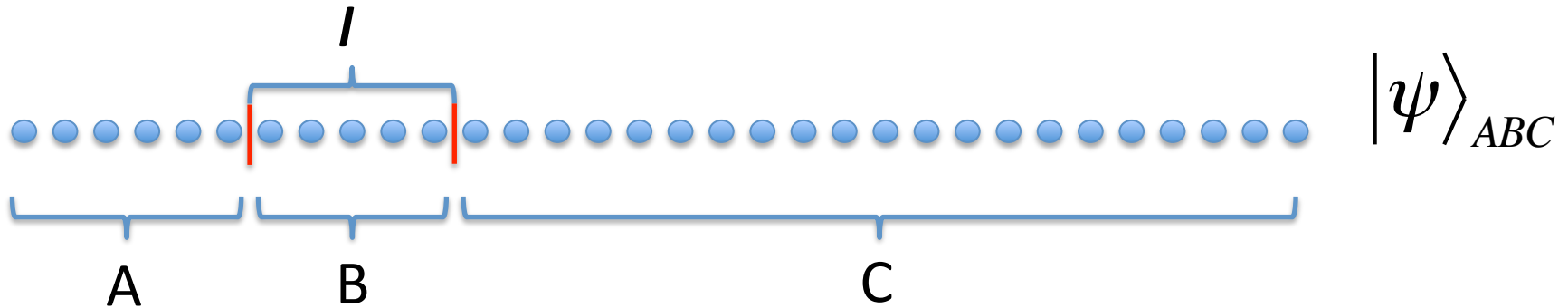
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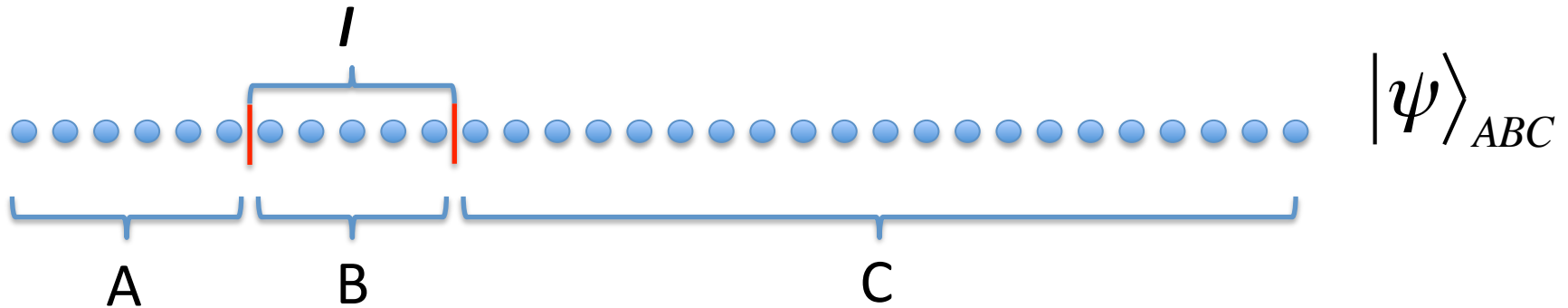
$$H_{\max}^{\nu}(C) \geq 3H_{\max}^{\delta}(B) \Rightarrow \text{Cor}(A : C) \geq \exp(-3H_{\max}^{\delta}(B))$$

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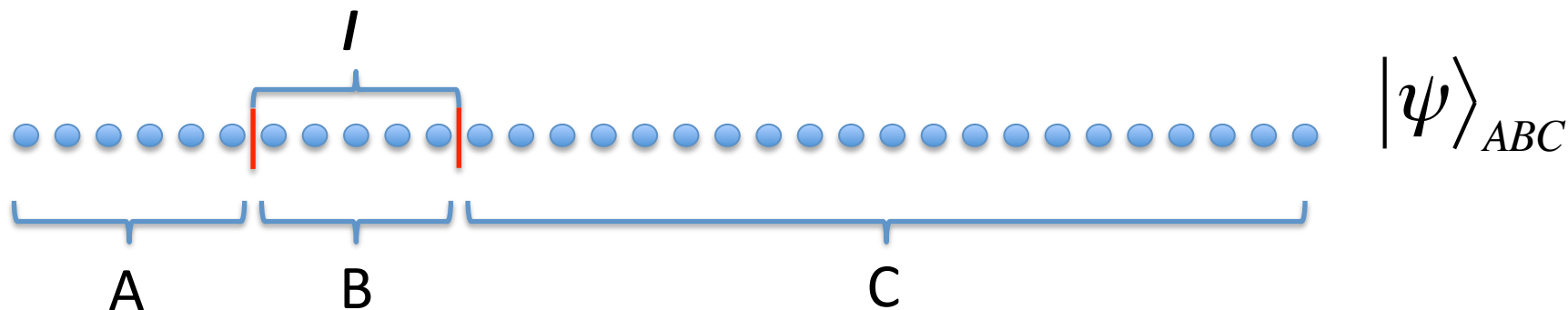


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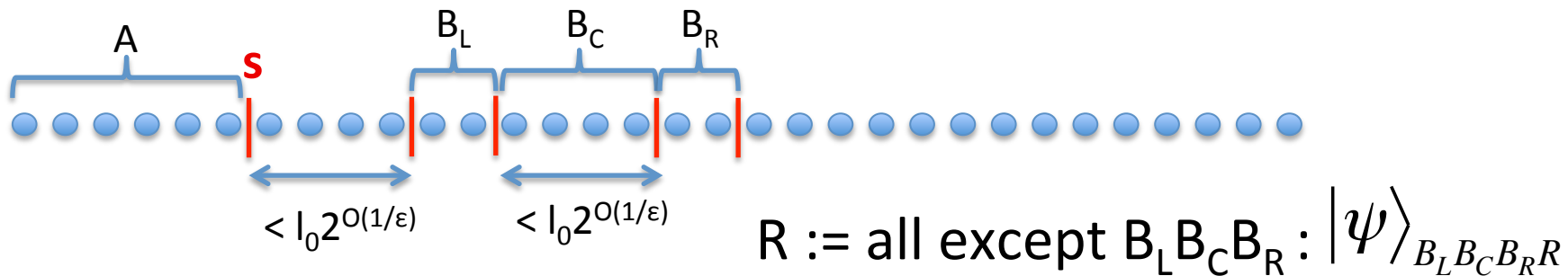
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Thus: $H_{\max}(C) < H_{\max}(B)$: **Area Law for C!**

It suffices to prove that nearby the boundary of C there is a region of size $< l_0 2^{O(\xi)}$ with *max* entropy $< l/(4\xi)$

Getting Subvolume Law (cheating)

“It suffices to prove that nearby the boundary of C there is a region of size $< l_0 2^{O(\xi)}$ with entropy $< I/(4\xi)$ ”



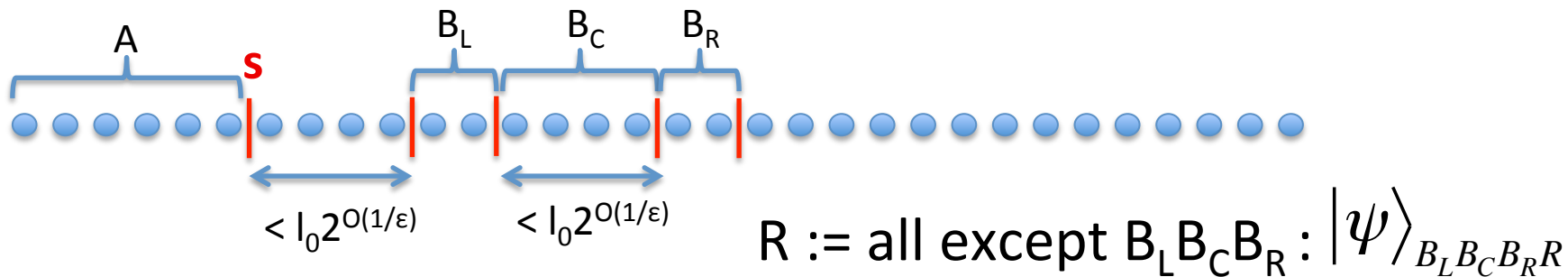
Hastings lemma with $\epsilon = 1/(4\xi)$, gives $I(B_C : B_L B_R) < I/(4\xi)$

Then from state merging protocol and ξ -EDC:

$$Cor(B_C : R) \leq 2^{-l/2\xi} \leq 2^{-I(B_C : B_L B_R)} \implies S(R) \leq S(B_L B_R)$$

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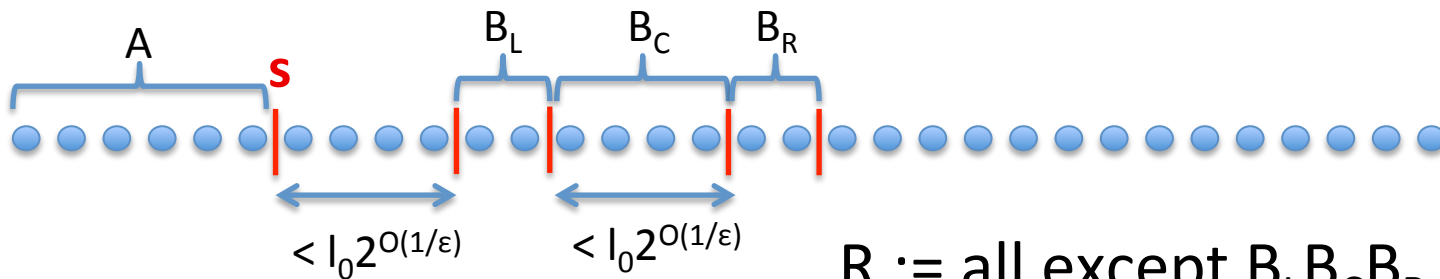
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$$Cor(B_C : R) \leq 2^{-l/2\xi} \leq 2^{-I(B_C : B_L B_R)} \implies S(R) \leq S(B_L B_R)$$

$$\text{Finally: } S(B_C) \leq S(B_C) + S(B_L B_R) - S(R) = I(B_C : B_L B_R) \leq I/(4\xi)$$

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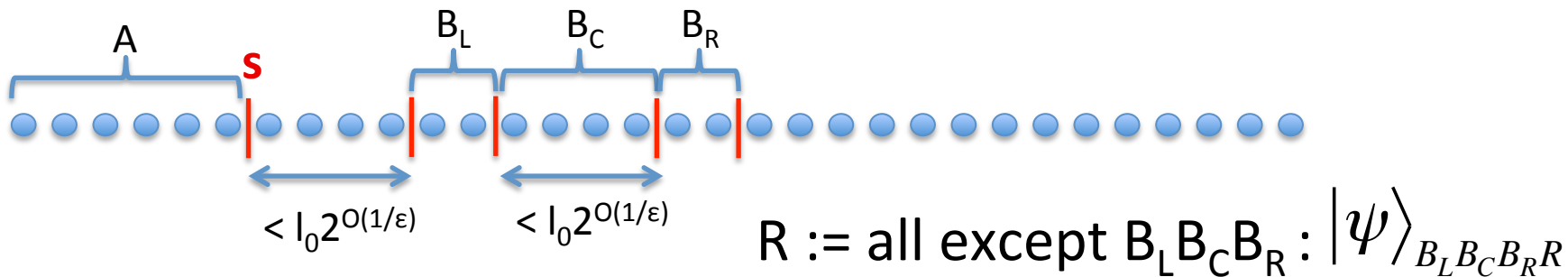
$R :=$ all except $B_L B_C B_R : |\psi\rangle_{B_L B_C B_R R}$

Single-shot Hastings lemma with $\epsilon = 1/(8\xi)$ gives

$$I_{\max}^{\delta}(B_C : B_L B_R) < 1/(2\xi)$$

Getting Subvolume Law

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Single-shot Hastings lemma with $\epsilon = 1/(8\xi)$ gives

$$I_{\max}^{\delta}(B_C : B_L B_R) < l/(2\xi)$$

Then from single-shot merging and ξ -EDC:

$$\text{Cor}(B_C : R) \leq 2^{-l/2\xi} \leq 2^{-I_{\max}^{\delta}(B_C : B_L B_R)} \implies H_{\max}^{\delta}(B_C | R) \leq 0$$

$$\text{Cor}(A : C) \leq \exp(-I_{\max}^{\delta}(A : B))$$

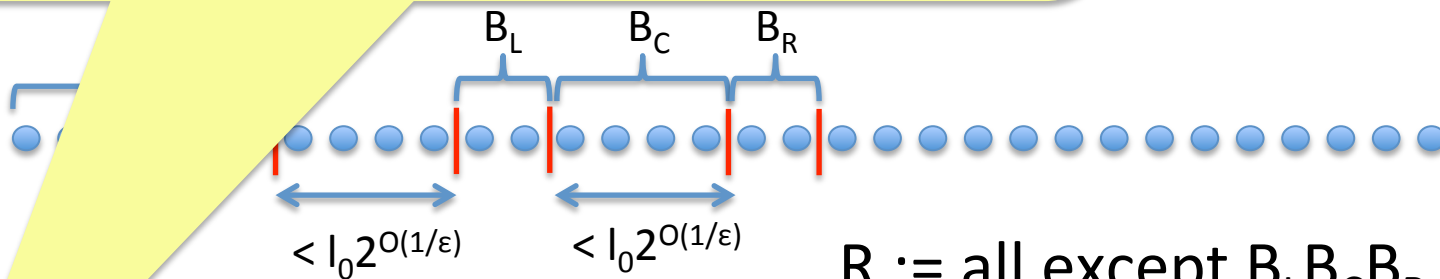
$$\Rightarrow H_{\max}^{\delta}(A | C) \geq 0$$

for $|\psi\rangle_{ABC}$

the Law

ary of C there is a

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$$I_{\max}^{\delta}(B_C : B_L B_R) < 1/(2\xi)$$

Then from single-shot merging and ξ -EDC:

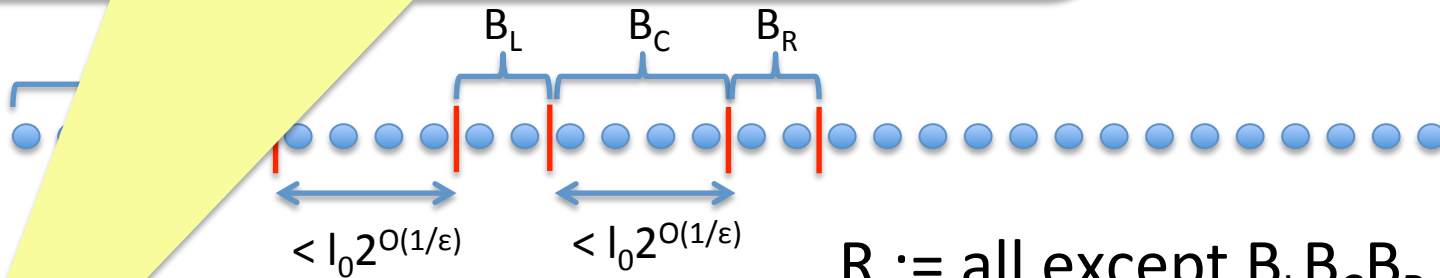
$$\text{Cor}(B_C : R) \leq 2^{-l/2\xi} \leq 2^{-I_{\max}^{\delta}(B_C : B_L B_R)} \Rightarrow H_{\max}^{\delta}(B_C | R) \leq 0$$

$$\text{Cor}(B_C : R) \leq \exp(-I_{\max}^{\delta}(B_C : B_L B_R)) \quad \text{the Law}$$

$$\Rightarrow H_{\max}^{\delta}(B_C | R) \geq 0$$

for $|\psi\rangle_{B_L B_C B_R R}$

any of C there is a
"



Single-shot Hastings lemma with $\epsilon = 1/(8\xi)$ gives

$$I_{\max}^{\delta}(B_C : B_L B_R) < 1/(2\xi)$$

Then from single-shot merging and ξ -EDC:

$$\text{Cor}(B_C : R) \leq 2^{-l/2\xi} \leq 2^{-I_{\max}^{\delta}(B_C : B_L B_R)} \Rightarrow H_{\max}^{\delta}(B_C | R) \geq 0$$

Getting Subvolume Law

$$\text{Cor}(B_C : R) \leq 2^{-l/2\xi} \leq 2^{-I_{\max}^{\delta}(B_C : B_L B_R)} \Rightarrow H_{\max}^{\delta}(B_C | R) \geq 0$$

$$\begin{aligned} H_{\max}^{\delta}(B_C) &\leq H_{\max}^{\delta}(B_C) + H_{\max}^{\delta}(B_C | R) \\ &= H_{\max}^{\delta}(B_C) - H_{\min}^{\delta}(B_C | B_L B_R) \\ &= I_{\max}^{\varepsilon}(B_C : B_L B_R) \leq l / (2\xi) \end{aligned}$$

By duality of min and max entropies
(Tomamichel, Colbeck, Renner '09)

Conclusions

- **EDC implies Area Law** in 1D.
- **Single-shot info theory useful in physics**, here condensed-matter physics/quantum many-body theory
- Single-shot is **more than to replace von Neumann by either min or max entropy: new regimes**
- Single-shot theory is **less structured**, but gives a **unified way of addressing different types of structures** (iid, permutation invariance, ergodicity, **exponential decay of correlations**)

Thanks!